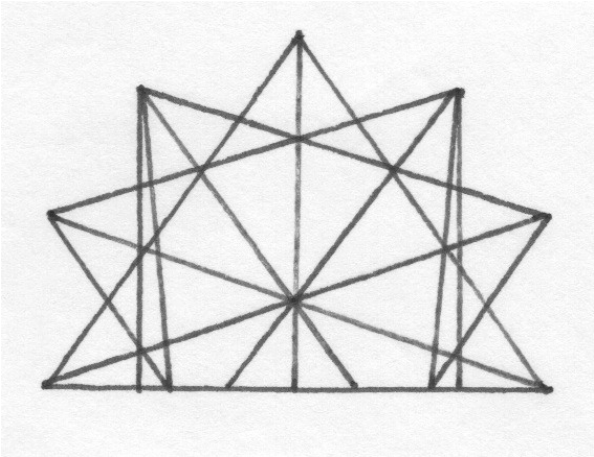
The Planetary Monochord



Siemen Terpstra

www.siementerpstra.com

THE PLANETARY MONOCHORD

Siemen Terpstra

ABSTRACT
The author examines various ancient proposals to assign the planetary system to a musical scale.

TABLE OF CONTENTS

1 Introduction
2 Disclaimer
2 The Old Geocentric Cosmology
4 Two Methodologies
4 The Auxiliary Diagrams
9 The Planetary Assignments
15 The Esoteric Ogdoad
18 Conclusion
Diagrams

INTRODUCTION

Mostly during the post Hellenistic Roman period, a group of writers suggested proposals for the particular musical scale that defines the "music of the spheres." They attributed the whole concept to Pythagoras, following Plato and Aristotle who also made the association with Pythagoreans. The majority of the ancient writers present their work in a vague manner, giving general indications but not ever specifying the monochord numbers. Some of the writers evidently know very little about musical tuning or monochord arithmetic. Others, such as Ptolemy, are tuning experts. When a writer does give some actual numbers, they form valuable clues to the intended division. This paper is a piece of speculative detective work, trying to eke out what particular harmonies are implied by the writers.

During the latter 1980's I accumulated a collection of diagrams (here numbered as **Figure 1** to **Figure 37**) relating to this issue. The heart of these diagrams sits in **Figures 28** to **33**. Here I have assembled the historical assignments of planets to scales. **Figure 34** is a special, summing-up diagram that puts the historical evidence within a wider context of possibilities. These diagrams are surrounded by what I will call 'Auxiliary Diagrams' that concern related issues—aspects of ancient Greek tuning theory, monochord arithmetic, and other topics. Most of these diagrams will get only brief comments, since I have already explained them elsewhere. This whole paper can be approached as an appendix to my two previous essays—on the tetrachord (*Homage to Ptolemy*) and on Canonics (*The Laws of Harmonics*). Whenever something cropped up relating to planets and monochords, I threw it into a separate folder. Now I'm finally writing the accompanying text.

The diagrams were not originally generated in any fixed or particular order. Nevertheless, I intend to make my commentaries on the diagrams more or less in the

given sequence, with various cross-references. However, before beginning with my commentaries, I need to set the stage and clear up a few conceptual muddles.

DISCLAIMER

I want to make it clear from the start that this paper is *not* an effort to support or reassemble the old concept of the 'harmony of the spheres.' As far as I'm concerned, the notion died with the death of geocentric astronomy and the 17th century rise of modern observational astronomy. Before that time many people rather blindly believed in it as a quasi-religious ideology closely related to astrology. Let us not pretend that it has some factual basis.

Moreover, this paper does not concern itself with or support the various modern efforts to redefine and reconstitute the concept. Such speculations are ongoing among a small circle, often related to various schemes to remake Pythagoras. I refer, for example, to 17th century Kepler's Planetary Music based on his inscribed polygons (crude fit) and his elliptically generated glissandi. Another example is the 18th century Titius-Bode 'Law' that achieves only a mediocre fit. The same can be said for the 20th century researcher Goldschmidt and his nodes of crystal growth. Since the Renaissance various occultists, neo-hermeticists like 17th century Robert Fludd, astrologers, Rosicrucians, Theosophists (Blavastsky), Anthroposophists (Steiner), esoteric circles like Gurdjieff, yoga theorists, New Agers, and more have professed some variant scheme where planets constitute a musical (generally scalar) order. However, without exception, all must admit only to an 'idealistic' search for norms that never quite match up to the real phenomenal world. People want to find a familiar harmony in the planets, but the reality is rather one of considerable complexity.

Instead, this paper discusses only the ancient Greek-Roman schemes of the planetary harmony. Inevitably, these proposals lie close to their contemporary music cultures. The majority of the writers imagined the harmony as a useable scale made up of two tetrachords within approximately an octave space. We cannot appreciate their perspectives without some grounding in tetrachord theory and monochords, since monochords formed the overwhelmingly dominant means by which the ancients explored and expressed musical tunings. The question always comes down to this: which monochord division is intended?

THE OLD GEOCENTRIC COSMOLOGY

According to long-standing ancient belief, the Earth rests immovable at the center of the cosmos. Around it spins eight concentric circular orbits or spheres that bear the various planets. These planets wheel around the earth, each in its own cycle. The cosmos is imagined to be finite in size, like an egg or an onion. Each sphere is visualized as a solid invisible crystal-like shell, harder than adamant yet more translucent than air. The outermost shell holds the fixed pattern of the stars—the Stellar Realm. It spins around the Earth from east to west (assuming a northern hemispherical perspective) every twenty-four hours, carrying with it the entire planetary system in its grip. Within this outer

bounding framework the 'wandering stars' including the Sun and Moon revolve more slowly in the opposite direction, inching their way around the zodiacal belt from west to east. Some move very quickly with relation to the stars, for example the Moon that completes its zodiacal circuit in about twenty-nine and a half days. Other planets move more slowly, notably Saturn, which takes roughly twenty-nine and a half years to go around the loop.

Each planet has its own independent orbital period and from this information the ancients speculated over the correct order of the shells. The most dominant sequence goes: Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, Stars. However, Mercury and Venus are always close to the Sun and each other. Sometimes these two planets switch places. One also finds the pattern: Moon, Sun, Venus, Mercury, Mars, and so on. Variants appear among different writers because the ancients had no reliable means to properly define the order of the planets or their distances from each other.

The paradigm always remained somewhat unstable, and there were skeptics throughout the ancient history. In the 4th century B.C. Heracleides suggested that Venus and Mercury orbit the Sun. In the same century Callippus, a student of Eudoxus, showed that we need at least 34 spheres in order to account for the planetary movements. In the 3rd century Aristarchus put forward an alternative heliocentric model. However, it never became popular and the 2nd century Seleucus became the last known astronomer to champion a heliocentric regime. The geocentric model would long endure. Nevertheless, the old model sat under some strain as astronomers made more careful observations and tried to save the appearances. The presence of many irregularities makes a reasonable explanation increasingly challenging. Since they assume that everything in the heavens must move in perfect circles, astronomers needed an increasingly complex gear-box mechanism of various epicycles (wheels within wheels) to account for retrogrades and so forth. The model became increasingly cumbersome, making a musical correspondence also more difficult and artificial. Yet for many hundreds of years, people continued to believe in the planetary music—a testimony to the inertial powers of religious memes.

Although most surviving references to the planetary harmony come from the Roman era, the concept evidently has older precedents. However, the early evidence is conflicted and peculiar. Plato gave *two* incompatible versions (**Figure 28**), both of which may have been intended as satirical parodies. Before that time the Pythagorean philosopher Philolaus professed a famously peculiar cosmology (according to Aristotle) that includes a counter-earth and a 'central fire' that is not the Sun. Aristotle's evidence may not be entirely trustworthy, since he also produced other questionable polemics against the Pythagoreans as a means of attacking Plato. Some scholars claim that Philolaus represents the true Pythagorean position, but we have no way of knowing. Pythagoras wrote nothing and became the repository of many conflicting theories. Unfortunately, the meager surviving fragments of Philolaus do not specify a particular planetary scale. In spite of the vagaries of the history, we can best assume the preponderance of the standard 'onion' model with the stationary Earth at the center and a normative planetary roster. It is this paradigm that was assigned a musical scale.

TWO METHODOLOGIES

The whole subject sits within two different conceptual frameworks. Let us call these two approaches *temporal* and *spacial*. Temporal first. One can reasonably base the harmony on the known periodic cycles of the various planets. Ratios between these orbits can be interpreted in a musical manner as vibratory ratios. So far we stand on solid ground. But when one looks at the numbers, the system displays much complexity—not a scale or a consonance, rather a hard dissonance. I have examined the temporal model in two previous papers. In *Harmonics in Ancient Calendrics* we see that setting up an accurate calendar has always been a rather tricky business. One must necessarily make some compromise between accuracy and usability, since the orbital interaction isn't simple. In my paper *The Octave Law* the notion of octave equivalence brings all of the planetary orbits into a representative octave. Using this rubric one can more easily see that the interaction is no recognizable scale, diatonic or otherwise.

Ancient astronomers, even before the Greek era, knew the periodic cycles of the planets. The 7th century Assyrians drew up ephemeride tables for use in astrology. This established information influenced speculations over the arrangement of the planets. However, oddly enough, the temporal concept was never invoked as a basis for the planetary scale, perhaps because it fails to make a simple scale. Instead, the ancient writers appealed to a *spacial conception*. The musical harmony comes from *the distances between the shells*. This conception is highly amenable to a monochord model; indeed, it may have come directly from it. Since the ancients had no proper means to measure the actual distances between the planetary shells, they felt free to invoke a simple or familiar harmony. Astronomers could then conveniently derive the cosmic arithmetic from the simple arithmetic of monochord division. Looking at the history of astrology-astronomy, the further we go back to its early roots the more prominent become various significant monochord numbers (such as 60, 72, 360 and so on). Music and astronomy must have co-evolved together. At any rate, during the Roman period and also during earlier times a monochord forms a fertile ground for a planetary system, but such schemes inevitably generate a fantasy.

THE AUXILIARY DIAGRAMS

Figure 1 and **Figure 2** present the two boundless sources of the rational ratios. The Sub-harmonics have priority due to the monochord environment. The ancients sometimes referred to the harmonics indirectly as 'sub-contrary,' the reciprocal of the monochord norm. From these two wells flow the products of the Musical Means (AM and HM). These two given interval triangles may be properly integrated into a single fundamental expression as the *Lambdoma*, **Figure 24**. Here both sides stand equally represented and the partnership with Silence is made visible. Here we have an open-ended hierarchical address system for any just ratio to the n-Limit.

Figure 3 shows the beginning of the ratio hierarchy subjected to size measurement by alternative resolutions. Back then I applied the concept of *variable resolution* only to equal temperaments (irrationals), but one can also make their fields

pliable and even rational. The term 'semitone' here obviously means diatonic or large semitone—in this selection of resolutions I didn't include a chromatic semitone version. The comma (c) and the schisma (s) prove highly valuable for measuring just intonation ratios, but back then I perhaps undervalued the clisma (k). On the diagram the solid line between ratios 3:4 and 1:5 indicates the first entrance of 5-Limit consonances. Similarly, between 5:6 and 1:7 the septimals enter, proceeding to 9:10. Here the table arbitrarily stops just short of the entrance of 11-Limit ratios, hence the title 'Tetractys series.' The dotted line between 7:8 and 1:9 displays the place where the 7-Limit consonances shade into soft dissonances, using as criteria the progressive clouding of difference tones. By the time one reaches a ratio such as 15:16 we have crossed over into the wide territory of hard dissonances. This method of generating a hierarchy among the musical ratios proves useful, but each ratio still presents a unique case with specific characteristics in its difference tones. Hence the specifics of the hierarchy need not become dogmatically rigid. Ratios are individuals with family relations.

The central theme of monochord work consists of making alternative divisions through the process of 'taking the mean' or 'following the middle path.' Its theoretical foundation rests in the *Progression of Means* that generate the *Genera*. Various diagrams in this collection bring out aspects of the Progression of Means, notably **Figure 4** and **Figure 5**.

Figure 4 sits squarely within the monochord environment of the Arithmetic Mean (AM). It shows how the sequence of the first and simplest divisions progressively generate 2-Limit, 3-Limit, 5-Limit, 7-Limit, and 11-Limit harmony. I have diagrammed each monochord division so that the reader can become comfortable with the notion of numerical monochord sequences. The mese number is emphasized in bold as a convenient shorthand. **Figure 4** also collects together various associations, symbols and mythical descriptions that relate to this division. I took the process as far as **6**:12 so that I could include the 'Mousike' and also the 11-Limit archetypes. This particular diagram inevitably betrays my 1980's bias toward mainstream (3 and 5-Limit) harmony. Since then I have come to re-evaluate the extent and the charms of the 11-Limit neutral intervals.

Figure 5 looks at the Progression of Means on the circular graph. It includes the Geometric Mean (GM) so that one can see how this irrational value mediates between the two Musical Means. Although one naturally associates the Progression of Means only with the just reciprocals of the AM and HM, irrationals cannot be entirely banished from our model, because they also have meaningful relations to the just ratios. Any theory of harmony that aims to be comprehensive needs to include both rationals and irrationals.

Figure 6 highlights the division **4**:8 or mese **4** for short, and its precursors of mese **2** and mese **1** outlined by Plato's disclosure of the Progression of Means. During the 1980's I called the division **4**:8 *the cosmological monochord* because it actualizes the five classical Elements and puts them into their proper density order. Now I prefer to call it *the elemental monochord*. The diagram shows the proper place of the element *aether* within early Greek philosophy (before it was re-defined by Aristotle). The early

philosophers used the poetic term *aether* sometimes as a stand-in for *air*, sometimes as an equivalent substitute for *fire*, and sometimes as an independent element that sits between *air* and *fire* in its density.

Figure 7 consists of a long (12 page) compendium of divisions that have some structural interest and relate to the planetary proposals. It suffers from the old problem of compilations. Looking back on it I am tempted to say: oh no! Maybe I should have included this one or that one. For instance, I could also have included the collection of historical divisions by Ptolemy, Archytas and others. But then, it was never intended to be exhaustive, and the historical monochords by Ptolemy and company can already be found in my paper on the tetrachord.

Figure 8 shows the important 5-Limit division of mese **72**. It has such prominence that I have named it *the planetary monochord* and titled this paper after it. However, I do not mean to imply that this division forms the *actual* planetary harmony. Instead, it wins the prize because its family garners the most references (direct and indirect) among the ancient writers. By the term 'family' I refer to the various subsets, the divisions of mese **36** and **18** and **9**. The harmony **72**:144 also has certain outstanding innate structural features that I have examined elsewhere. For all of these reasons I treat it as a sort-of 'default position' when one desires a diatonic ogdoad scale. It also exhibits a natural partnership with a special 3-Limit comma-shifted cousin, the division **1152**:2304. Ancient tuners always had a choice between 3-Limit and 5-Limit paradigms of the diatonic scale.

The harmony **72**:144 and its 3-Limit variant given by Euclid have such prominence that several more diagrams are devoted to it. In **Figure 23** the two harmonies are presented on the circular graph so that one can observe the subtle but easily perceptible differences between them. A comma is not a very large interval, too small to form an independent scalar element, but it nevertheless alters the character of the scale quite profoundly. From this diagram we also see that both versions are symmetrical—the axis of symmetry shown in the dotted line. **Figure 27** also concerns symmetry and follows the growth of the single-strand and double-strand diatonic harmonies. All of the 3-Limit matrix patterns exhibit symmetry, but some of the 5-Limit structures instead form non-symmetrical reciprocal pairs. For some ancient (and modern) theorists, the presence of symmetry enhances status.

Also related to the **72**:144 diatonic ogdoad, **Figure 22** shows the four most consonant modes of both the 3-Limit and the 5-Limit archetypes. They form two pairs through their Duals. These harmonies have great intrinsic structural significance and sometimes crop up in ancient references. At the bottom of **Figure 8** we see a famous instance. Following on, **Figure 9** makes more explicit Plato's 3-Limit 'world-soul' monochord construction with its outcome the mese division **1536** and implied mese **96**. Although this construction was not intended to display a planetary assignment, it could be used for one. Any harmonic ogdoad can hold eight shells between the octave.

Figure 10 illustrates how some prominent symmetrical harmonies can be generated by expanding a single-strand and a triple-strand matrix structure. Here the 3-Limit 'Mousike' harmony becomes the 3-Limit pentad structure ruled by the momentous division **72**:144. We can carry on in this fashion to give the symmetrical heptad ruled by **432**:864 and the ennead ruled by the division **5184**:10368. In the 5-Limit triple-strand structures, the core chromatic pattern ruled by **60**:120 expands out into the 'calendrical series' ruled by **360**:720. One can continue on to generate **4320**:8640 and **51840**:103680 at the schisma boundary. All of these divisions (and some closely related ones) have relevance to ancient cosmology and the concept of a Great Year—a grand division that harmonizes all of the lesser cycles.

Figure 11 explains the underlying rationale of the triangular-hexagonal matrix, an obvious and suitable vehicle for recording 3-Limit and 5-Limit tuning operations. An integration of the 7-Limit requires yet another dimension, as pointed out by Plato in his progression: point, line, surface (triangle), solid (tetrahedron). However, one can make a simulation of it within the 5-Limit matrix by using a clisma-shifted substitution. This 7-Limit substitution field is presented in **Figure 17**. The world of 7-Limit harmony is best approached as an extension of the 5-Limit mainstream, what I sometimes call 'major-minor' harmony. Only with the 11-Limit do we make a radical breakthrough into a new world of neutral (neither major nor minor) intervals.

Figure 11 also shows how the traditional language of *perfect*, *medial* and *imperfect consonances* can be based upon the 'consonance hexagon.' Of course, we can also ground it in the hierarchical ratio order shown in **Figure 3**. The 7-Limit also has its own consonances, but they are distinctive enough to be called 'septimals' and in a subtle manner merge into soft dissonances. At the bottom of the page, **Figure 11** displays my old conception of the genera, to be given more explicit detail in **Figure 13**. Back then I tied it closely to the matrix and took the classical genera (diatonic, chromatic, enharmonic) as a closed concept. Although the reference to the matrix is entirely understandable, the Progression of Means forms a more elegant and consistent foundation. Moreover, it becomes open-ended and makes a firm basis for the valuable concept of *variable resolution.* The Progression of Means itself emanates from the sequence of epimoric ratios, fundamental pillars of the architecture of harmony.

Figure 12 again shows my favored resolutions: schisma, comma, enharmonic and diatonic semitone. Emphasis on the comma resolution 'flag' demonstrates its suitability for an old theme of ancient Greek philosophy: a city-state is like a harmony and vice versa. The center-point houses the statue of the goddess. The consonance hexagon corresponds to the sacred precinct. The region within the comma boundary represents the 'downtown' of the city, where three sorts of citizens live: naturals, sharps and flats. To the west and the east lie the 'suburbs' defined by comma shift. The city wall has four gates: east and west (schisma related) as well as north and south (clisma related). The citizens of this comma-resolved community must pay a price, a compromise (temperament) that creates the city wall and banishes the micro-intervals of the schisma and clisma.

musical pitches, or the order of the planets, or the distances between the shells. I have filled in the planetary order that Plato specifies in the *Republic* scheme, but they don't necessarily coincide. The story in the *Phaedrus* gives little away. Aristotle was nevertheless quite influenced by the 'elements inside planets' scheme. He divides the cosmos into two zones: a sub-lunar zone where four material substances hold sway and a celestial planetary zone where *aether* rules. Aristotle redefined *aether* as a common ground or substrate for the whole cosmos, a support for four physical primary elements as substances here on Earth and a stellar perfection to be found in the heavens. Due to Aristotle's influence, the 'sub-lunar' theory had a long life.

The scheme in *Republic* proves to be even more peculiar. Here Earth has been banished from its normative mese position in the middle, to be replaced by Mercury; indeed, Earth has been eliminated entirely. According to the rationale, Earth has no musical pitch because it sits immovable. The scheme no longer poses the normative octave between Earth and Heaven *(fire)*—here it sits between the Moon and Heaven. The position of the octave has shifted. Rather than the normative placement between the middle and the periphery, it now surrounds the mese in two disjunct diatonic tetrachords. Last but not least, Plato has also reversed the direction of the planetary sequence.

Both schemes appear within an elaborate mythological literary context. Plato invented many fanciful stories, yet he himself expressed ambivalence over their value. At best they form only likely tales or 'semblances.' At worst they are dangerous lies, so odious that he would banish the poets from his model city. Later in the *Phaedrus* he warns us to be skeptical over the written word altogether. Aristotle claimed that Plato had secret doctrines not written down at all but passed on orally. He also expressed frustration that Plato often does not say what he means or mean what he says. Plato himself admits that he commonly uses both 'comedy' and 'tragedy.' Often he is not serious, but rather building some elaborate parody with polemical intent. These false facades are usually directed at the early Greek philosophers. He should be treated as a complex satirist, not as a straightforward one-dimensional 'serious' philosopher.

Plato never specified numbers for his *Republic* planetary harmony, but he left the valuable clue that the tetrachords are diatonic. I have chosen the most harmonious exemplar (having the smallest monochord numbers) for both the 3-Limit and 5-Limit scales. As always, I use the pitch 'C' as mese. However, most scholars are used to the choice of 'A' as mese, following the authority of the late Roman writer Boethius. Consequently, I have shown both on the diagrams. The 'A' notation can be converted to Boethius' practice by simply ignoring the comma-shift signs. My notation is tied strictly to function and thus it must be sensitive to comma alteration. Boethius' notation isn't strictly functional in origin; rather, it comes from position in the line of fifths. It also assumes that only 3-Limit harmony has value or even existence. 5-Limit harmony is out.

Whatever Plato's original intentions, the upcoming Roman writers took Plato as a serious authority. Their formulations often reproduce features found in the *Republic* scheme. Plato's influence can be clearly seen in the scheme of Nicomachus, **Figure 29**. The two diatonic tetrachords still surround the mese, though now they are conjunct. Earth

is still banished, but Nicomachus has taken the further step of eliminating the Stars (Heaven) from the harmony. Consequently, it now covers less than the normative octave. He has returned the scalar direction to the norm, but, as if to hedge his bets, he also proposed a reversal back to Plato's version. He also shows ambivalence in the placement of Mercury and Venus. By reducing the number of participants to seven, the mese position becomes invariant to scale direction. In Nicomachus the mese is happily occupied by the Sun. As usual, no monochord numbers are given. However, the most harmonious division in this situation proves to be mese **48** for the 5-Limit harmony and mese **768** for the 3-Limit version. These numbers take us back to Plato's 'world-soul' monochord division.

Figure 29 also features a modern alteration of Nicomachus' scheme. Here the two tetrachords are taken as disjunct, returning us to the monochord numbers of Plato's *Republic* division. In order to maintain the range found in Nicomachus, one more element of the scale (the trite) must be eliminated. The reformed scale is like the Platonic version but with the planetary direction reversed. I have included this modern example in order to highlight the arbitrary nature of this game. One has a choice over conjunct or disjunct tetrachords, over the placement of the octave, and over the direction of the planetary order. Selections of these independent parameters generate many alternative schemes, but none has ultimate priority of the others. Even the norm that the Earth sits in the mese position and the Stars (Heaven) on the open string only arises when we consider a model consistent with the monochord environment. Many Roman era writers do not feel constrained by monochord realities and most probably had little experience with actual monochord arithmetic. Nicomachus himself claims to be a 'Pythagorean expert' but his book uses many words to say very little of substance.

So far our tetrachords have all been of the diatonic variety. Of course, it need not be the case. The boundary tones of a tetrachord remain fixed, but the two inner tones are allowed to be flexible. According to the size of the inherent intervals, a tetrachord can be diatonic, chromatic, or enharmonic and various 'shades' between these norms. Not surprisingly, a sizable group of writers have opted for a chromatic tetrachord, even hinting that an enharmonic tetrachord is also acceptable. Their chromatic proposal appears on **Figure 30**. Like in Plato, we still have two disjunct tetrachords oriented around the mese, but now both the Earth and the Stars are allowed back into the harmony. We still see the reversed planetary direction given in Plato. This approach forms essentially a variant of Plato's *Republic* scheme, only using chromatic tetrachords rather than diatonic. In effect, the harmony now coalesces around the consonance hexagon. The version below by Achilles Tatios changes the scale slightly and alters the order of the planets. One can proliferate such variants endlessly.

Thus far the two tetrachords surround the mese position, following Plato. **Figure 31** returns the scheme to the normative space between the mese and the open string—the monochord's model octave. Cicero's version still echoes Plato, since he goes back to diatonic tetrachords and the reversed planetary direction. Like Plato, he omits Earth from the harmony. He differs from Plato in using conjunct tetrachords, in keeping with the

theorists' standard model. Had he used disjunct tetrachords he could have integrated the 'Mousike' sequence.

Dio Cassius presents the same arrangement but the planetary direction is reversed. Here at last a writer comes quite close to the normative 'planetary monochord' shown on **Figure 8**. The only difference lies in the reduced need for a heptad rather than an ogdoad, since Earth is excluded. As shown below, Dio Cassius also relates the order of the days of the week to the 3-Limit line of fifths. Such fanciful schemes based on free association have proliferated though history.

The great scientist Ptolemy supported a novel approach. Abandoning the octave scale, he assigned the planets to the fixed tetrachord frames, **Figure 32**. In an unusual move, he expresses the number sequence using *harmonic numbers* rather than the usual monochord numbers. In this way he demonstrates his mastery of musical arithmetic, as well as an awareness of the reciprocals. Perhaps he also intended a little test that would confound the uninitiated novice. We are alerted to this musical game by the extra tone on top—monochords by convention cover no more than two octaves. By adding the extra tone he creates the conditions where the monochord (falling) sequence is almost identical to the rising (harmonic) scale. Both sequences use numbers from 8 to 36 and they are *almost* the same. The only discrepancy concerns the 'F' position of the Lesser Perfect tetrachord. Ptolemy has highlighted this position by using a number with a fraction in his rising sequence. It turns out that the falling sequence must also use a number with a fraction, but a different number! The reason that this situation occurs lies in the structure of the 3-Limit Tetrad, which exhibits axis symmetry but not point symmetry. Had Ptolemy used a harmony with point symmetry he could have made the reciprocals absolutely identical between the octave, but here they have a real discrepancy. If a tuner follows his given number sequence but mistakenly interprets it as a monochord sequence, he will run into trouble.

Anatolius does just that. In addition, he also introduces some copy errors into Ptolemy's numbers. We see here that Anatolius is just a naïve copyist who has little understanding of the underlying basis of the numbers. Perhaps he has no real interest in the numbers themselves—he wants mainly to fill the nine positions with the planets in their usual order. He may have been dissatisfied with Ptolemy's assignment, which is certainly original and unusual. Ptolemy wanted the 'elements within planets' scheme (12 positions) that originated in Plato's *Phaedrus*. He has managed to take the twelve actors and squeeze them into his nine positions by doubling up on three shells. Now two planets combine in the mese position and the four elements reduce into two groups, a 'dense' pair and a 'rare' pair. The whole construction shows wonderful imagination.

Ptolemy's number game between the reciprocals shows mastery of his craft combined with a certain devious delight in puzzles. This sort-of deliberate obscurity pervades many texts on musical tuning throughout history, both in the east and west.

Ptolemy left a school of tuning theory that dominated the high Arabic culture of the 9th, 10th, and 11th century Iraq. **Figure 33** shows two planetary assignments from this

Islamic world. In al-Kindi's effort, part of the two octave tetrachord frame is filled in to make a complex scale that covers less than an octave. He has deviated pretty far from Ptolemy's scheme. In the assignment by the 'Brethren of Purity' we come back to Ptolemy's number sequence (8 to 36). Here it is treated as a monochord but it includes numbers from *both* the upward and downward sequence. In addition, key numbers are deliberately modified so that the scale now has a 7-Limit and a 13-Limit element. This inclusion of higher prime numbers conforms to the spirit of Ptolemy, who famously sanctioned n-Limit harmony.

Both monochords use the normative planetary direction. Al-Kindi's assignment reminds us of Dio Cassius in **Figure 31**, where Earth is eliminated. The monochord by the Brethren is more like that of Ptolemy, where the two element shells remind us of Plato's *Phaedrus*. Oddly enough, the open string has no assignment. Perhaps this scheme was preferable because it puts the Sun in the mese position—a popular move already seen in Anatolius (**figure 32**), Pliny, Martianus, Censorinus, Theon (**Figure 30**), and Nicomachus (**Figure 29**).

These Arabic assignments conclude my survey of ancient geocentric schemes. The 9th century Irish monk John Scotus Eriugena envisioned a half-way house between a geocentric and a heliocentric cosmos. He kept Earth in the center, with four shells—that of the Moon, Sun, Saturn, and Stars. The Sun shell owns it own grand epicycle of the planets Mercury, Venus, Mars and Jupiter. Consequently, these planets must either sing the same tone as the Sun, or else some tones related to the Sun. It appears that Eriugena disregards any connection with a monochord and simply equates the geocentric Earth, Sun, Saturn and Stars shells with various octaves of the same tone. The other planets that circle the Sun make movable tones depending on where they sit in their orbits. This scheme shows great imagination and considerable complexity, but it takes us away from the ancient notion of a simple fixed planetary scale.

Setting aside Ptolemy as a unique and special case, the ancient assignments fall into two camps. The first group uses two diatonic tetrachords to make a diatonic octave scale. In this group we find Plato, Nicomachus, Cicero and Dio Cassius. The other group uses two chromatic tetrachords to generate a more complex octave scale that embodies chromatic aspects. In this group sits Pliny, Martianus, Censorinus, Theon and Achilles Tatios. Within this group one also finds hints that enharmonic tetrachords are also permissible. I have chosen not to draw up an enharmonic sequence, because ancient records indicate that enharmonic intervals were not popular in the music culture of the Roman era—except among theorists. Whether or not this is true remains controversial, but nevertheless the sweep of history in the west favored diatonic octaves. The chromatic structures lost their foothold as tetrachord theory eventually disappeared. Assignments since the Renaissance have been almost exclusively diatonic. Hence one can speak of a diatonic mainstream in the west.

In **Figure 34** (which is five pages long) I have examined various modes of the 3-Limit and 5-Limit diatonic heptads, as a resource for alternative diatonic scales on a monochord. As noted elsewhere, the three most consonant 5-Limit diatonic scales sit in

the divisions of mese **24**, mese **30**, and mese **36**. This set is now further augmented with mese **45**, mese **32**, mese **27**, and mese **40**. The number sequences are expressed both as a monochord pattern and a harmonic series pattern. The suggested alternative planetary assignments either place the Sun in the mese position or else use the octave sequence that eliminates the Earth from the harmony. In this diagram the historical 'likely story' sits within a wider realm of possibilities. At the end of the diagram the harmonic heptad (the consonance hexagon) serves as an alternative to the diatonic heptad. Of course, one could also choose any number of different harmonies as a basis for this fantasy.

Figure 35 shows that a single number sequence can yield all three of the most consonant 5-Limit diatonic scales; mese **24**, mese **30**, and mese **36**. The very simplicity of the inherent numbers promotes the conjecture that it must have been known at an early age. The elegance of this graph is meant to make a fitting conclusion to this study. But then…later I added **Figure 36** and **Figure 37**. These diagrams add a little additional information.

Figure 36 clears up confusion concerning the names of the seven classical Greek modes. We moderns use the names given to us by Glareanus after 1547. But he had mixed up the original Greek names. For example, the mode that we call 'Phrygian' was called by the Greeks 'Dorian,' and so on. The two modes marked by asterisks were the most popular classical modes, according to the theorists. On the left, I have marked various prominent monochord divisions that make up these modes. However, Greek musical culture eventually changed so that only one tuning, (**36**:72) need serve for all the modes by the use of 'octave species.' The history of Greek and Roman tuning culture is decidedly complex, and I have tried to steer clear of the particulars.

Figure 37 demonstrates the official Christian Church modes, showing the poverty of the system in comparison to the old complexity of classical culture. The only tuning allowed is the 3-Limit Heptad, and not even all of the inherent modes are used. The theorists also recognized the Ogdoad **1152**:2304, reproducing the old tuning given by Euclid. It was expressed as the line of fifth Bb, F, C, G, D, A, E, and B. This scale was applied to the original medieval keyboard that had one 'raised' key (the Bb). It became the official basis for European music right up until the 14th century. However, in the Middle East and India the old pluralistic subtleties lived on. Moreover, in the west, various regional musics survived that did not comply with the rigid rules of the church theorists. It remains an open question how reliably the theorists faithfully reflected the actual practices of working musicians.

In summary, the very unreality of the 'onion' model of eight shells lets it become a mirror for the current musical preferences. During the ancient period that means monochord scales based upon tetrachords. Ptolemy's advanced approach of n-Limit harmony lived on for a time among the Islamic scientists, but in the west the scientific flame eventually dampened, due to theological repression. Only the 3-Limit diatonic harmony remains among late Roman theorists. At the end of the ancient period, the 6th century Roman Boethius passed it on to the Middle Ages. However, during the 2nd century heyday of the ancient writers (the time of Ptolemy) the tuning culture exhibited

gnostic writings. Indeed, he made the famous dictum that *'the path upward and the path downward are one and the same'* and its companion *'the beginning and the end are common on a circle.'* Yet more of his enigmatic sayings can be interpreted as support for a gnostic perspective. However, space does not allow me to pursue the topic in depth here. I refer the reader to my work on early Greek philosophy.

We should not forget that musical tuning itself also has an esoteric component. It manifests itself in the subject of temperament—a perennially controversial issue. What status do irrational ratios have? The theorems of Thales that generate the GM provide a sure key to the irrational division of a monochord. As explained elsewhere, one particular family of divisions is uniquely easy to implement using classical geometrical methods. Happily, it also has benign structural characteristics that make it suitable as a musical framework. I refer to the system of 72 *parts* (72-et) made explicit in Aristoxenus. He probably intended it as a scientific means to gauge the metrical properties of intervals, analogous to the modern system of 1200 *cents* (1200-et). With this tool he simulated the imaginative 7-Limit harmonies of Archytas and laid out various 'shades' of the genera. Aristoxenus had a small group of followers and a school that persisted for hundreds of years. Nevertheless, he has been overwhelmingly vilified throughout history for daring to favor irrational ratios. By the close of the ancient period his work was buried and misunderstood. It went underground, which only enhances its esoteric status.

The recognition of an atomistic gauge to measure interval size was a great achievement of Aristoxenus, but he also presents something even greater. He held a clear notion of variable resolution. For 72-et can be practically conceived as quarter-tone (24-et) with inflections. Again, it can be looked at as a deeper resolution of 12-et. Moreover, Aristoxenus suggested that if the 72 parts do not provide an adequate resolution for a desired context it is entirely permissible to divide the part itself in half, pointing to a deeper resolution of 144-et. In other words, the continuum between the octave space can be resolved to whatever level is desirable. For Aristoxenus the family of resolutions 12-et, 24-et, 36-et, 72-et, and 144-et form signposts along a path with no fixed ending. Thus he provides a practical answer to an old philosophical question that animated early Greek philosophy: how can we best map the continuum of infinite divisibility?

By an extra-ordinary synchronicity, the family of irrational divisions dominated by 72-et employs the same numbers found in the old arithmetical planetary monochord of the octave **72**:144 and **36**:72. This convergence enhances the esoteric status of these numbers in the field of musical tuning.

While looking for the origins of the association between a musical scale and an ascension through seven steps, I came to the realization that it must be very old. We already see it hidden in the old Babylonian story of the descent of Ishtar into the underworld. She passes through seven gates before reaching the Goddess of the infernal realm. Here one sees a reversal—a descent rather than an ascent, but such reversals only announce the esoteric undercurrents. The path upward and downward are one and the same. Many tales of reaching the underworld by entering the cave, like the story of

Orpheus, have similar esoteric implications. Such tales obscurely point to a gnostic perspective.

In the Jewish bible one finds the story of Jacob. He lays down to sleep and dreams of a ladder that reaches from Earth to Heaven. On it various angels both ascend and descend. Our very word for a musical scale comes from the Latin *scala*, which means 'ladder.' In most Indo-european languages, the term for a musical scale translates as a 'tone-ladder.' It appears that the association between a musical scale and an ascension is quite ancient.

CONCLUSION

The notion of a musical planetary harmony had great longevity, in spite of the utter lack of any foundation for it. Why? Three interconnecting factors bolstered its status. First of all, the great prestige of Pythagoras and Plato assured its widespread diffusion. Secondly, its occult associations gave it a certain power. On the other hand, we should not over emphasize this factor, since gnostics always formed rather small elites within society. Among most people a less educated and cruder religious philosophy held sway. The third and possibly the most critical factor must be the prestige of the old and engrained astro-religions in the ancient period. Even serious scientists like Ptolemy make essentially no distinction between astronomy and astrology. They assume that astrology has validity. The prevalence of this mind-set creates the environment where such fantasies can flourish.

-completed in June, 2016, Amsterdam

Figure 1: The Harmonics

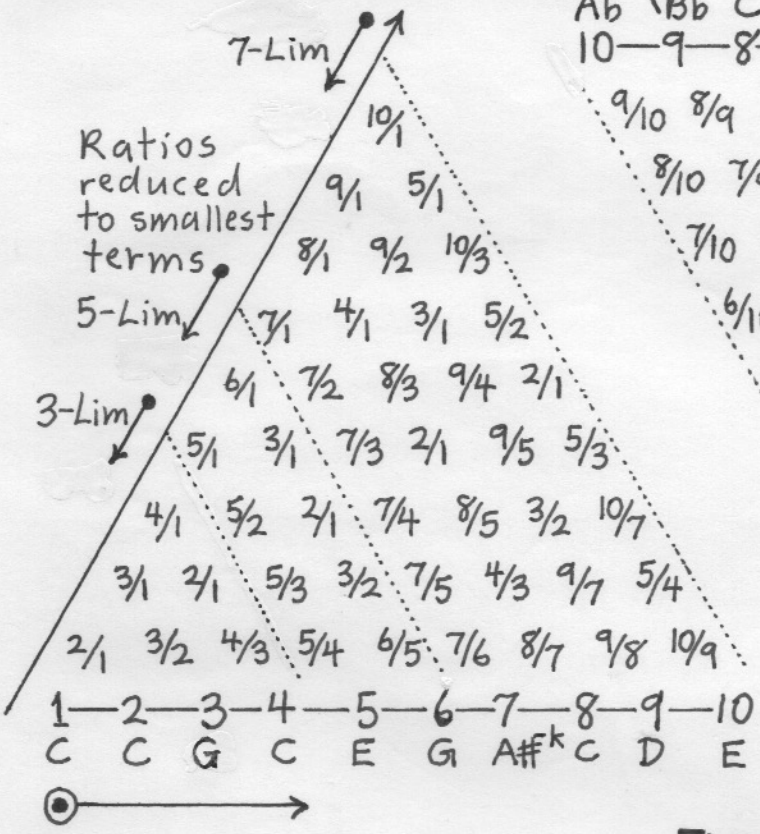


Figure 2: The Sub-harmonics

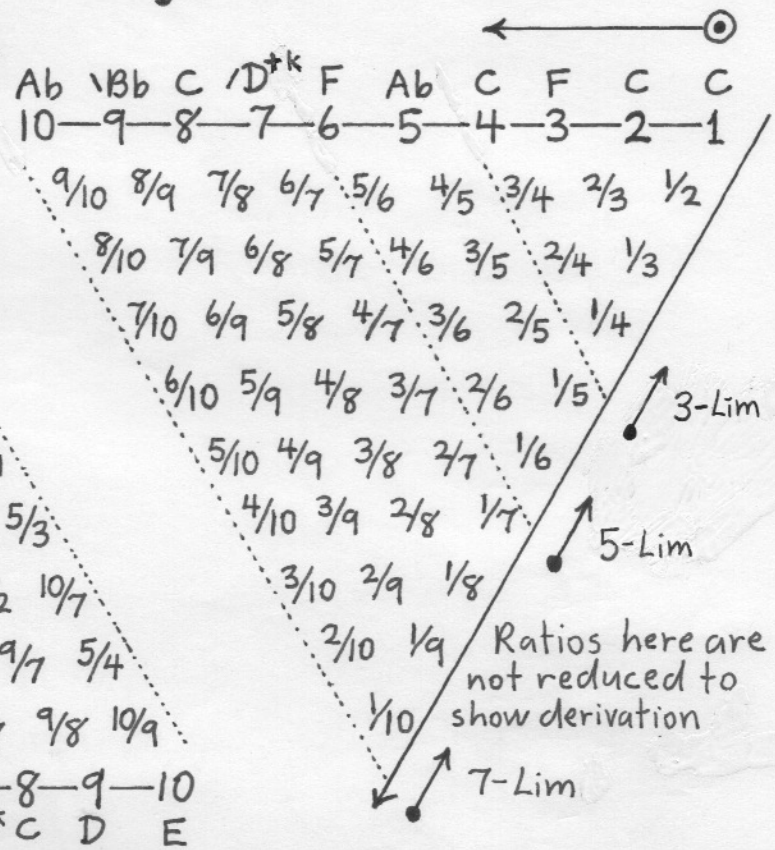


Figure 3: Tetractys series

The Musical Ratio Archetypes: Metrical properties

R = Ratio S = schisma C = comma e = enharmonic Se = semitone

⊙ = Octave

R	S	C	e	Se
1:1	0	0	0	0
1:2	⊙=612	⊙=53	⊙=31	⊙=12
1:3	⊙+358	⊙+31	⊙+18	⊙+7
2:3	358	31	18	7
1:4	2⊙	2⊙	2⊙	2⊙
3:4	254	22	13	5
1:5	2⊙+197	2⊙+17	2⊙+10	2⊙+4
2:5	⊙+197	⊙+17	⊙+10	⊙+4
3:5	451	39	23	9
4:5	197	17	10	4
1:6	2⊙+358	2⊙+31	2⊙+18	2⊙+7
5:6	161	14	8	3
1:7	2⊙+494	2⊙+43	2⊙+25	2⊙+10
2:7	⊙+494	⊙+43	⊙+25	⊙+10
3:7	⊙+136	⊙+12	⊙+7	⊙+3
4:7	494	43	25	10
5:7	297	26	15	6

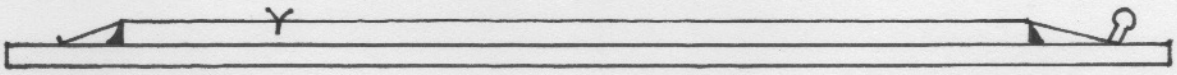
R	S	C	e	Se
5:7	297	26	15	6
6:7	136	12	7	3
1:8	3⊙	3⊙	3⊙	3⊙
3:8	⊙+254	⊙+22	⊙+13	⊙+5
5:8	415	36	21	8
7:8	118	10	6	2
1:9	3⊙+104	3⊙+9	3⊙+5	3⊙+2
2:9	2⊙+104	2⊙+9	2⊙+5	2⊙+2
4:9	⊙+104	⊙+9	⊙+5	⊙+2
5:9	519	45	26	10
7:9	222	19	11	4
8:9	104	9	5	2
1:10	3⊙+197	3⊙+17	3⊙+10	3⊙+4
3:10	⊙+451	⊙+39	⊙+23	⊙+9
7:10	315	27	16	6
9:10	93	8	5	2

Figure 4: A close look at the first six operations of the Arithmetical Proportion – the basis for the ancient 'sacred' monochord science. The first four operations define the **Elements** with its Cosmological Monochord, as well as Plato's progression of 'dimensions': point, line, plane, and solid. Thus the Matrix of relations (a record of tuning operations) is established. Each operation is emphasized by a large number at right. This number defines the Mese in the falling monochord scale, and the base number for the rising Harmonic Series scale. Both Harmonic (H) and Monochord sequences (M) are given. The Mese number is always highlighted for easy reference. In addition, some of the symbolism arising from each division-operation is outlined.

Before performing any operation with the movable bridge, we are given the monochord's open string and its reference pitch (here chosen 'C'). It has two aspects:

A – Unsounded. The Silence (0), the unmanifested state, the primeval waters, the Abyss, the Deep, the preliminary Chaos, the Nun, the Apsu. **0**

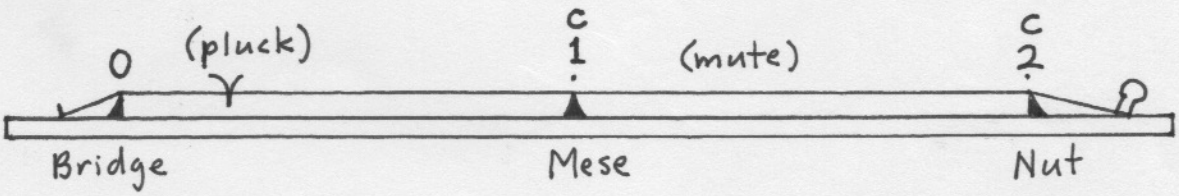
B – Sounded. Plucking the open string reveals THE ONE, THE MONAD, the first Element Heaven or Fire, the Androgyne, the high god, the first manifest vibratory event, the Source, Unity, the potential All (present within the Harmonic Series), the reference pitch, the Unchanging, the stellar realm, Zeus, Amon, Hermes, Thoth, Brahman, Anu.



1

[1] The 1st operation: Divide the string in half, placing the movable bridge in the middle (Mese). Now 1:1 → 1:2. In the pebble arithmetic, place the 'mother' bead to begin the Matrix of relations. Emanation: 2, THE DYAD, the Element Earth, the goddess, the Mese, the Point, the Double, the Octave (microcosm within macrocosm), Cyclical Identity, the two Realms (Harmonics and Sub-harmonics), rising and falling, the Reciprocal, the Other within the One, the Intelligible Realm, the Intellectual Principle, the Matrix of relations, the 1st Logos (1:2), the spiral of the changing, Yin-Yang, Tao, Ṛta, Maāt Self-dualing.

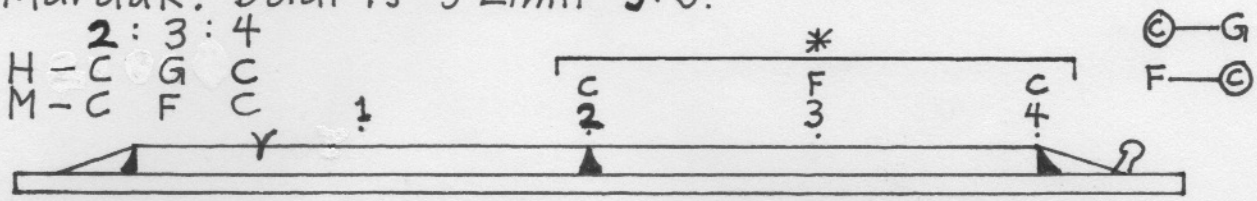
	1	: 2
H –	C	C
M –	C	C



©

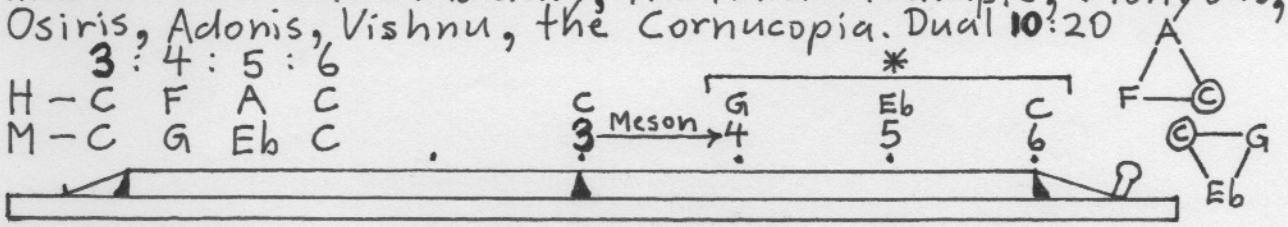
2

[2] The 2nd operation: Divide the octave (1:2) in half (2:3:4) yielding the archetypes of 3-Limit harmony, the musical fifth (2:3) and fourth (3:4). Place a pebble to form a horizontal line, the characteristic of 3-Limit tuning. Emanation: 3, THE TRIAD, the Element Air, the male god, the hero, the demiurge, the Other manifest, diversity, the Tetrachord frames, the Line, the Arrow, the marraige (of 2+3), the Soul-Principle, the musical intervals of power, the Perfect Consonances, Apollo, Shu, Shiva, Marduk. Dual is 3-Limit **3**:6.



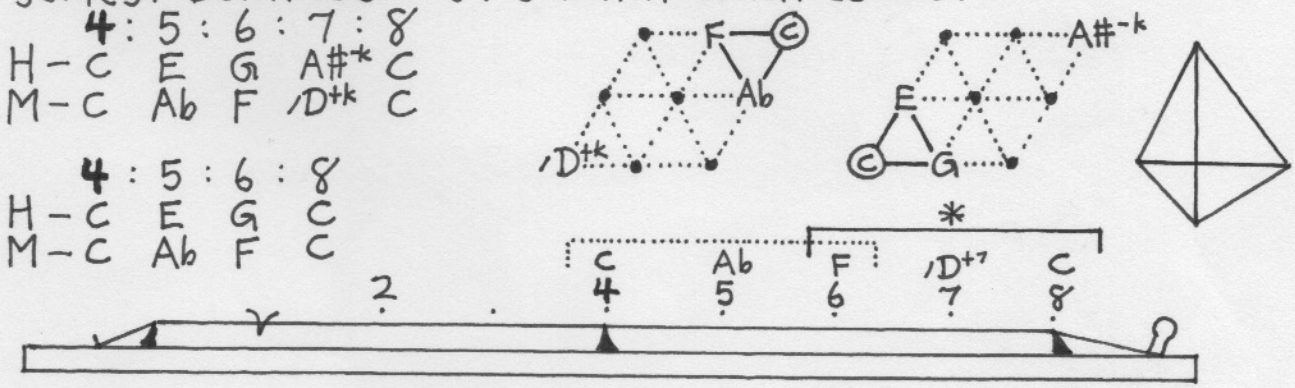
3

[3] The 3rd operation: Divide the musical fifth (2:3) in half (4:5:6) to yield the archetypes of 5-Limit harmony, the thirds (4:5 and 5:6). Place a pebble to form a triangle. Emanation: 5, THE PENTAD, the Element Water, the Surface, the triangulated web of triads, the child god, the Senarius, the Four Elements or Roots, the trinity of powers (2, 3, 5) which are masks of the One, the Meson Tetrachord, the musical intervals of Beauty, the Nature-Principle, Dionysus, Osiris, Adonis, Vishnu, the Cornucopia. Dual **10**:20



4

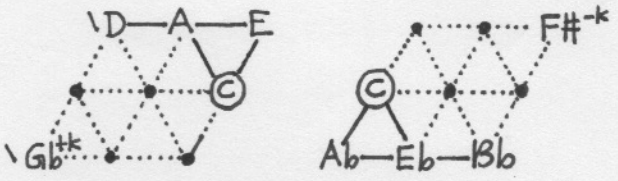
[4] The 4th operation: Divide the musical fourth (3:4) in half (6:7:8) to yield the 7-Limit archetypes 6:7 and 7:8. In the pebble arithmetic this forms the 3rd dimension (Tetrahedron), although a surface 'substitute' can be found. Emanation: 7, THE HEPTAD, the esoteric Element Aether, the Five Elements, the mystery god, the Solid, the completed series. Dual **105**:210. 5-Limit dual **15**:30.



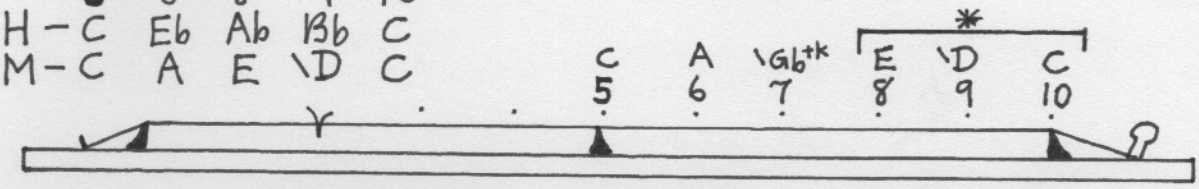
5

5 The 5th operation: Divide the musical major third (4:5) in half (8:9:10) to yield the two wholetones of the Just Intonation, 8:9 and 9:10. Since nine is not a prime number, no new 'dimension' or Element is emanated. This operation shows why operation 4 is a natural completion point for the Cosmological Monochord sequence. However, this division gives the 7-Limit series known as the Tetractys sequence, comprising all the ratios up to 10. 7-Limit dual is **252**:504. The dual of the 5-Limit subset is **36**:72.

	5	: 6	: 7	: 8	: 9	: 10
H –	C	Eb	F#$^{-k}$	Ab	Bb	C
M –	C	A	\Gb^{+k}	E	\D	C



	5	: 6	: 8	: 9	: 10
H –	C	Eb	Ab	Bb	C
M –	C	A	E	\D	C



6

6 The 6th operation: Divide the musical minor third (5:6) in half (10:11:12) to yield Ptolemy's 'exotic' 'three-quarter' tones 10:11 and 11:12. Pebble placement is problematic, though possible at the Antipodes of the Matrix. Emanation: **11**, THE UNDECAD. There is no classical Element assigned to it. It is exotic and foreign to many music cultures. It is usually avoided (7 also!). This division is used to illustrate that 3-Limit and 5-Limit subsets are the norm.

3-Limit	**6**	: 8	: 9	: 12		
	H –	C	F	G	C	
	M –	C	G	F	C	

The 3-Limit subset is the 'Mousike' or musical proportion, the first self-dualing harmony (symmetrical about the tonic).

5-Limit	**6**	: 8	: 9	: 10	: 12	
	H –	C	F	G	A	C
	M –	C	G	F	Eb	C

A core segment of the diatonic scale, subset of the cosmological heptad.

7-Limit	**6**	: 7	: 8	: 9	: 10	: 12	
	H –	C	D#$^{-k}$	F	G	A	C
	M –	C	Bbb^{+k}	G	F	Eb	C

A 7-Limit pentad scale whose dual is **210**:420

11-Limit	**6**	: 7	: 8	: 9	: 10	: 11	: 12	
	H –	C	D#$^{-k}$	F	G	A	Bb	C
	M –	C	Bbb^{+k}	G	F	Eb	Db	C

Dual is **2310**:4620

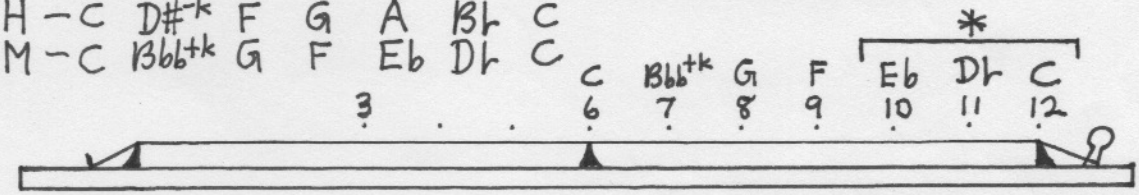
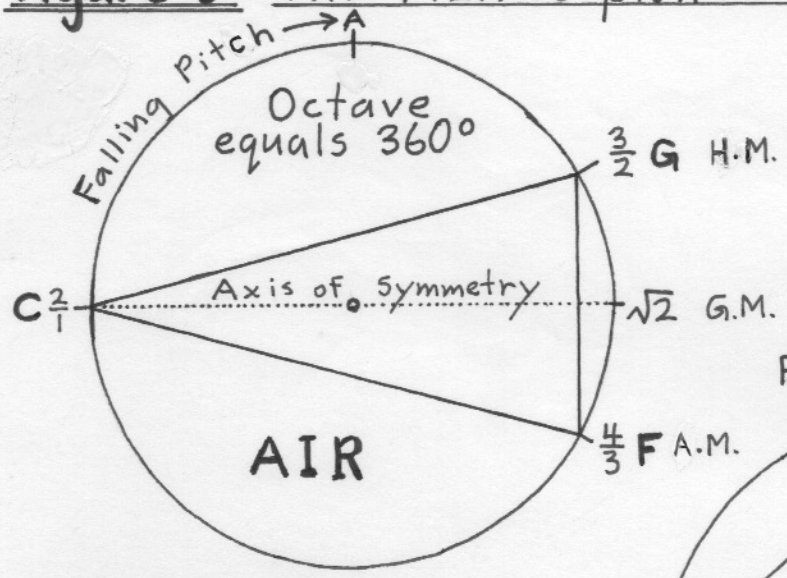
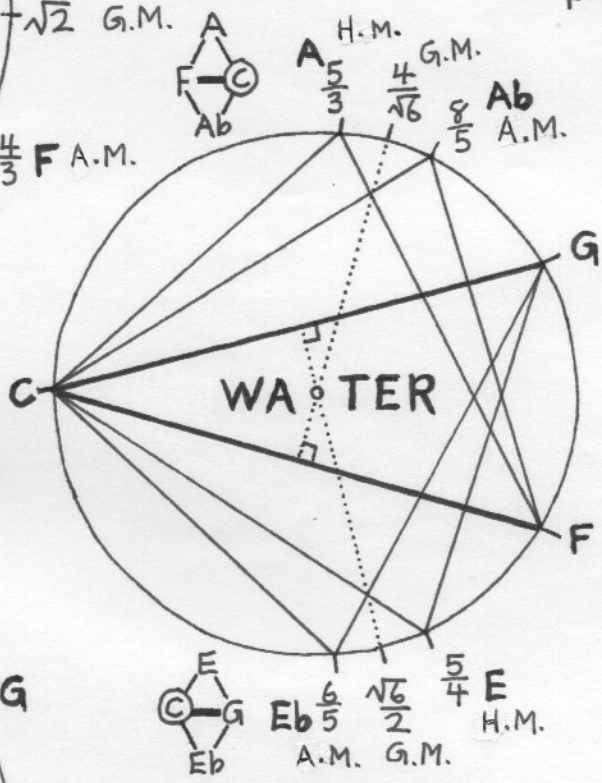


Figure 5: THE MEANS plotted on the Circular Graph:

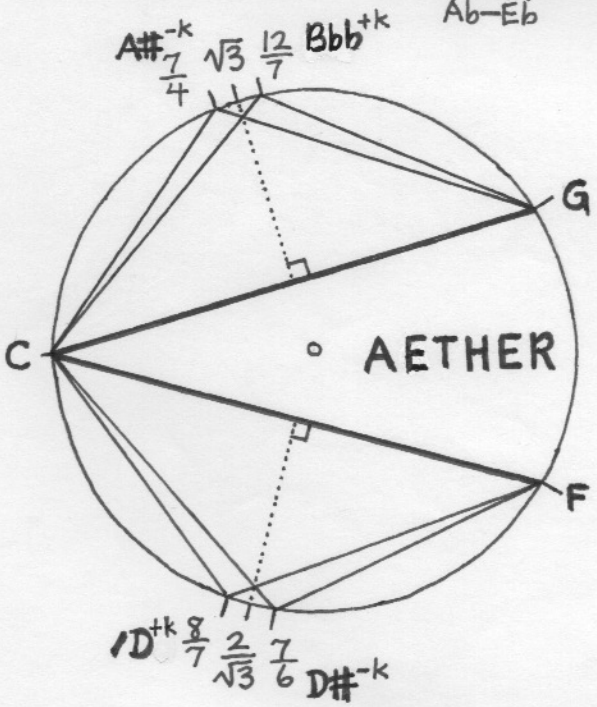


The Means of the octave yield the Fifth and fourth, hence 3-Limit harmony

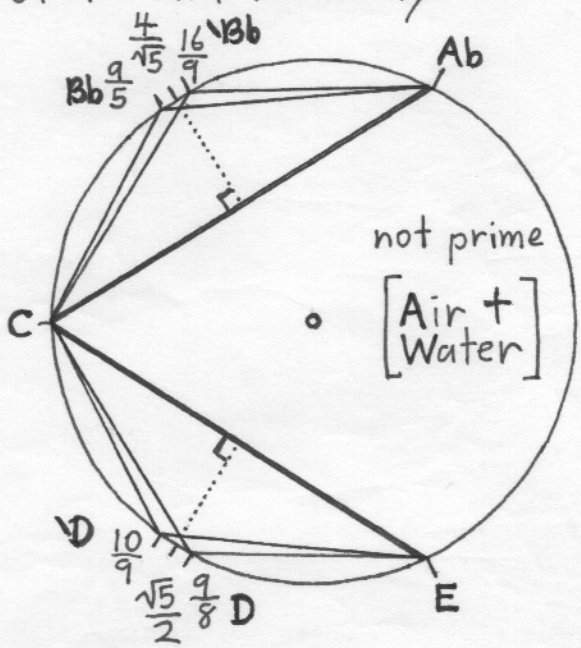
F—Ⓒ—G Ⓒ—G F—Ⓒ



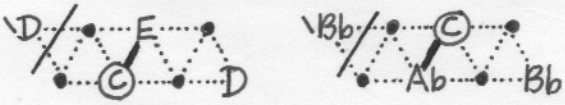
The Means of the fifths C-G and F-C yield the archetypes of 5-Limit harmony



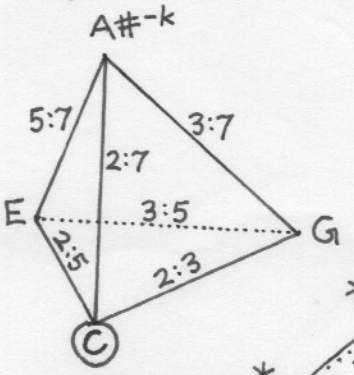
The Means of the fourths C-F and G-C yield the central ratios of 7-Limit harmony



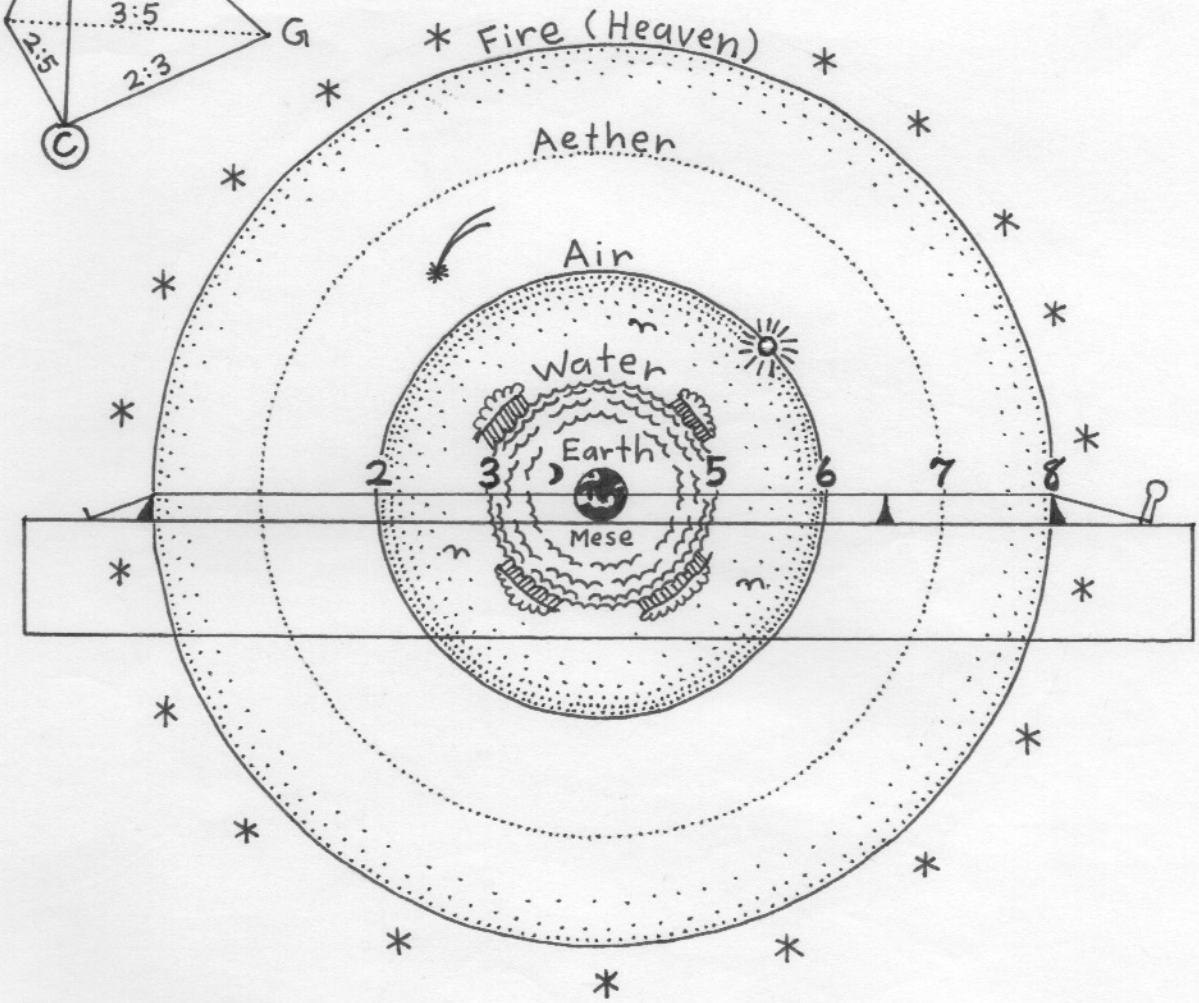
The Means of the major thirds C-E and Ab-C yield the two sizes of wholetones which characterize 5-Limit harmony



1:2
2:4
Figure 6 : The Cosmological Monochord **4:8**



4 : 5 : 6 : 7 : 8
C E G A#-k C
C Ab F /D+k C



Precis of the process forming the Cosmological Monochord

				pebble arithmetic		family
THE MONAD	1:**1**		(Heaven) Fire			
THE DYAD	[1] 1:1	⟶ 1:**2**	Earth	⊙	Point	2-Limit
THE TRIAD	[2] 1:2	⟶ 2:**3**:4	Air	⊙—●	Line	3-Limit
THE PENTAD	[3] 2:3	⟶ 4:**5**:6	Water	△	Surface	5-Limit
THE HEPTAD	[4] 3:4	⟶ 6:**7**:8	Aether	◇	Solid	7-Limit

Figure 7: An inventory of monochord divisions associated with the Cosmological Monochord and the various Planetary monochords. A brief descriptive commentary is given, the sequence in harmonics (H) as well as the monochord (M), and the matrix pebble pattern as well. The large number on the right indicates the "Emanation" number, which is the Mese of the monochord sequence and the base number of the harmonics sequence. It is hoped that this layout makes any relevant division easy to locate and reference.

The only 2-Limit sequence, an elemental monochord, establishing the octave foundation, the parent of every other sequence, self-dual: **1**

	1	: 2
H –	C	C
M –	C	C

Ⓒ

The elemental 3-Limit monochord, the parent by factors of 2 of **4**:8, **8**:16, **16**:32 and so on, whose dual is 3-Limit **3**:6. **2**

	2	: 3	: 4
H –	C	G	C
M –	C	F	C

F—Ⓒ Ⓒ—G

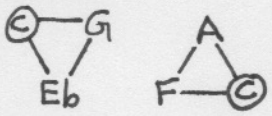
As the 3-Limit, it is the dual of **2**:4 and the source of the Meson Tetrachord **3**:4 : **3**

	3	: 4	: 6
H –	C	F	C
M –	C	G	C

Ⓒ—G F—Ⓒ

As the 5-Limit, it is an elemental monochord and the core of the tri-axial matrix, establishing the Just Triad, parent of monochords **6**:12, **12**:24, **24**:48, **48**:96, **96**:192 and so on. Its 5-Limit dual is **10**:20.

	– **3**	: 4	: 5	: 6
H –	C	F	A	C
M –	C	G	Eb	C



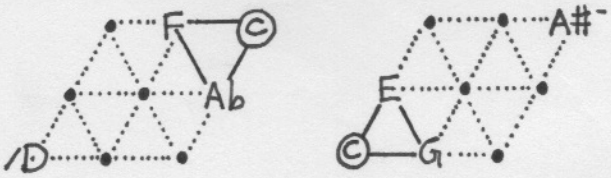
4

As the 5-Limit, it is a most harmonius position of the Just Triad, whose 5-Limit dual is **15**:30.

	4	: 5	: 6	: 8
H –	C	E	G	C
M –	C	Ab	F	C

As the 7-Limit, it is an elemental monochord, the last of the classical elemental monochords, dual 7-Limit **105**:210

	4	: 5	: 6	: 7	: 8
H –	C	E	G	A#⁻	C
M –	C	Ab	F	/D⁺	C

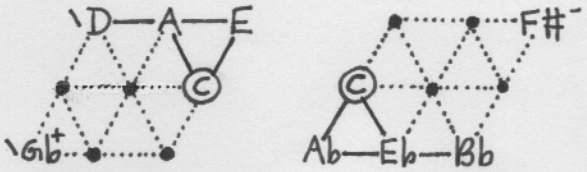


5

As the 5-Limit, it establishes the wholetones of Just Intonation, its dual is **36**:72

	5	6	8	9	10
H –	C	Eb	Ab	Bb	C
M –	C	A	E	\D	C

As the 7-Limit, it completes the Tetractys Series of ratios, whose dual is 7-Limit **252**:504



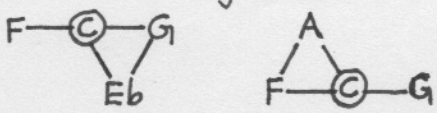
	5	6	7	8	9	10
H –	C	Eb	F#-	Ab	Bb	C
M –	C	A	\Gb+	E	\D	C

6

As the 3-Limit, it is the Mousikē, the "musical proportion," establishing the perfect consonances and the 3-Limit triad which become tetrachord frames. It is symmetrical about the generator and self-duals.

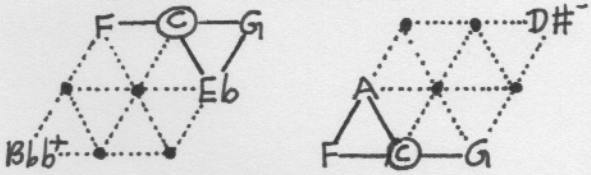
	6	8	9	12
H –	C	F	G	C
M –	C	G	F	C

As the 5-Limit, it forms the essential core of the diatonic scale, its dual being **30**:60



	6	8	9	10	12
H –	C	F	G	A	C
M –	C	G	F	Eb	C

As the 7-Limit, it forms a core 7-Limit scale, and duals with 7-Limit **210**:420



	6	7	8	9	10	12
H –	C	D#-	F	G	A	C
M –	C	Bbb+	G	F	Eb	C

As the 11-Limit, it establishes the '3/4th tones' of Ptolemy's Diatonic Hemiolon Tetrachord (**9**:10:11:12). Dual is the 11-Limit division **2310**:4620. Notation and mapping on the matrix are problematic.

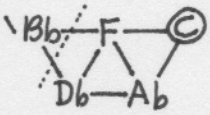
	6	7	8	9	10	11	12
H –	C	D#-	F	G	A	B♭-	C
M –	C	Bbb+	G	F	Eb	D♭-	C

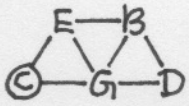
8

As the 3-Limit, it establishes the tetrachord frames for the Greek Greater Perfect system in Harmonics, used by Ptolemy for his 'Planetary Music', dual **9**:18

	8	9	12	16
H –	C	D	G	C
M –	C	\Bb	F	C

As the 5-Limit, it establishes the double-strand Pentad in its smallest numbers. Dual is **45**:90

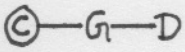


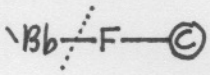


	8	9	10	12	15	16
H –	C	D	E	G	B	C
M –	C	\Bb	Ab	F	Db	C

9

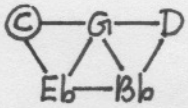
As the 3-Limit, it establishes the tetrachord Frames for the Greek Greater Perfect System, and is the parent monochord for the cosmological monochords **18**:36, **36**:72 and **72**:144. Its dual is 3-Limit **8**:16.

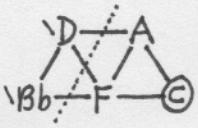




	9	12	16	18
H –	C	F	\Bb	C
M –	C	G	D	C

As the 5-Limit, it forms an essential Pentad subset of the Diatonic scales. Dual is **40**:80

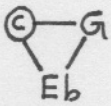




	9	10	12	15	16	18
H –	C	\D	F	A	\Bb	C
M –	C	Bb	G	Eb	D	C

10

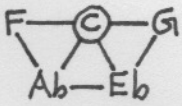
As the 5-Limit, it is the dual of monochord **3**:6
It establishes the minor triad in harmonics and the major triad on the monochord.

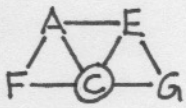


	10	12	15	20
H –	C	Eb	G	C
M –	C	A	F	C

12

As the 5-Limit, it establishes the most harmonius mode of the Pentad. The dual is **30**:60

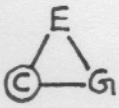




	12	15	16	18	20	24
H –	C	E	F	G	A	C
M –	C	Ab	G	F	Eb	C

15

It is the 5-Limit dual of monochord **4**:8



	15	20	24	30
H –	C	F	Ab	C
M –	C	G	E	C

Also as the 5-Limit, it forms the Diatonic Tetrachord in the standard Meson position. Dual is **24**:48

	15	16	18	20	(30)
H –	C	Db	Eb	F	C
M –	C	B	A	G	C

16

As the 3-Limit, the Tetrad in smallest numbers, used by Plato in Timaeus (circle of Different), whose dual is **27**:54

	16	18	24	27	32
H	C	D	G	A	C
M	C	Bb	F	Eb	C

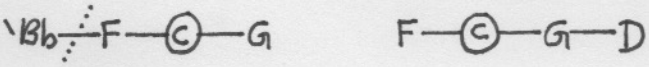
18

As the 3-Limit, a highly significant harmony placing the Tetrad in its most harmonius position on the monochord, thus establishing on the monochord the 'immovable' frames of the combined Greater Perfect and Lesser Perfect Systems. It is used by Anatolius to set the planets to the tetrachord frames. The same harmony is expressed in harmonics numbers by Ptolemy, achieved by using the dual – **24**:48. This symmetrical harmony forms the 'common ground' between the 3-Limit Diatonic Ogdoad and the 5-Limit Diatonic Ogdoad.

	18	24	27	32	36
H	C	F	G	Bb	C
M	C	G	F	D	C

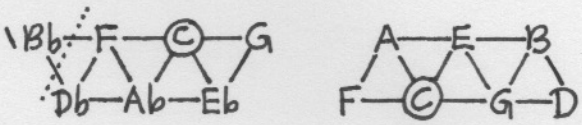
24

As the 3-Limit, the dual of 3-Limit **18**:36, it sets the Tetrachord Frames using harmonics, thus used by Ptolemy for Planetary attribution



	24	27	32	36	48
H	C	D	F	G	C
M	C	Bb	G	F	C

As the 5-Limit, it defines the double-strand Diatonic Heptad in smallest numbers. It is used for Planetary attribution by Nicomachus. The dual is **90**:180

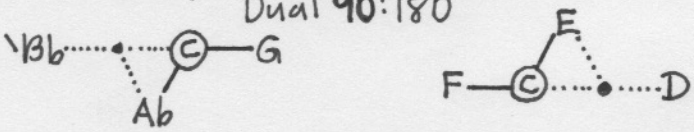


	24	27	30	32	36	40	45	48
H	C	D	E	F	G	A	B	C
M	C	Bb	Ab	G	F	Eb	Db	C

As the 5-Limit, it is the dual of the Diatonic Tetrachord **15**:30. For position on the Meson, see **36**:72

	(24)	36	40	45	48
H	C	G	A	B	C
M	C	F	Eb	Db	C

As 5-Limit, it makes the beautiful Didymus Diatonic Tetrachord Dual **90**:180



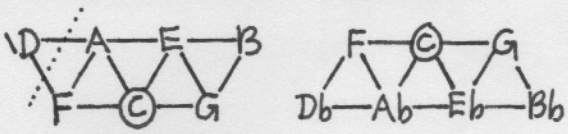
	24	27	30	32
H	C	D	E	F
M	C	Bb	Ab	G

As the 3-Limit, the dual of monochord **16**:32

	27	32	36	48	54
H –	C	Eb	F	Bb	C
M –	C	A	G	D	C

30

A very significant 5-Limit harmony associated with Babylonian cosmology, a harmonius Diatonic Heptad whose dual is **72**:144

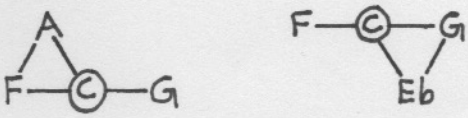


	30	32	36	40	45	48	54	60
H –	C	Db	Eb	F	G	Ab	Bb	C
M –	C	B	A	G	F	E	D	C

As the 5-Limit dual of the Pentad **12**:24

	30	36	40	45	48	60
H –	C	Eb	F	G	Ab	C
M –	C	A	G	F	E	C

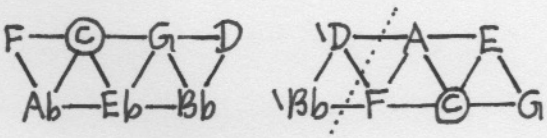
Also as the 5-Limit dual of the Tetrad **6**:12



	30	36	40	45	60
H –	C	Eb	F	G	C
M –	C	A	G	F	C

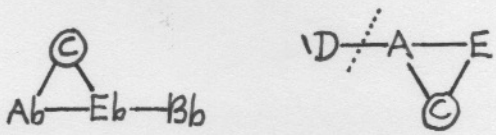
36

The most significant 5-Limit Diatonic Heptad, used for 'Planetary Music' more than the other Heptads, referred to by Plato, Nicomachus, Ptolemy, Dio Cassius, Plutarch, Boethius. Dual is **120**:240



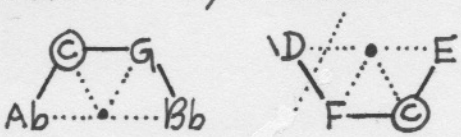
	36	40	45	48	54	60	64	72
H –	C	D	E	F	G	A	Bb	C
M –	C	Bb	Ab	G	F	Eb	D	C

It also defines the dual of 5-Limit Tetrad **5**:10



	36	40	45	60	72
H –	C	D	E	A	C
M –	C	Bb	Ab	Eb	C

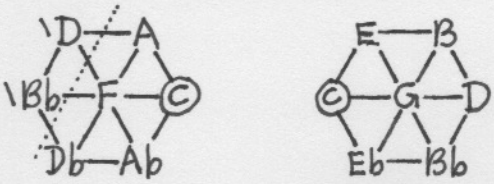
It also positions the consonant Diatonic Tetrachord in the Meson (see **24**:48), where it is named Ptolemy's Diatonic Syntonon



	36	40	45	48	(72)
H –	C	D	E	F	C
M –	C	Bb	Ab	G	C

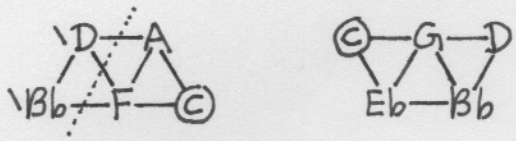
40

As the 5-Limit, it forms the Chromatic Heptad in its smallest numbers. Its dual is **45**:90



	40	45	48	50	60	72	75	80
H –	C	D	Eb	E	G	Bb	B	C
M –	C	\Bb	A	Ab	F	\D	Db	C

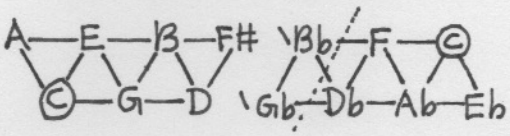
As the 5-Limit, it is the dual of Pentad **9**:18



	40	45	48	60	72	80
H –	C	D	Eb	G	Bb	C
M –	C	\Bb	A	F	\D	C

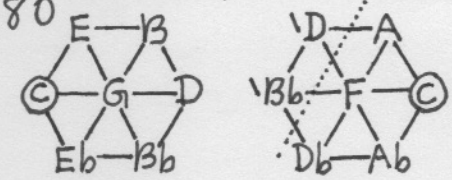
45

As the 5-Limit, it forms a most beautiful mode of the Diatonic Heptad on the monochord. Dual is **96**:192. It also presents the 1st possible appearance of the Syntonic Comma ratio 80:81, the extra element shown in brackets



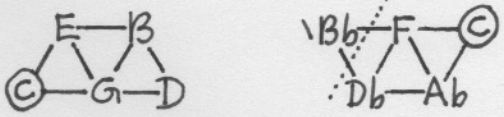
	45	48	54	60	64	72	80	(81)	90
H –	C	Db	Eb	F	\Gb	Ab	\Bb	(Bb)	C
M –	C	B	A	G	F#	E	D	(\D)	C

As the 5-Limit, it is the dual of the Chromatic Heptad **40**:80



	45	48	50	60	72	75	80	90
H –	C	Db	\D	F	Ab	A	\Bb	C
M –	C	B	Bb	G	E	Eb	D	C

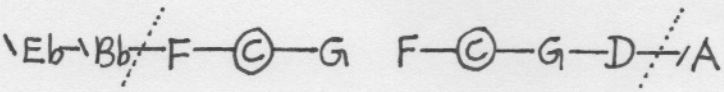
It is also the dual of the 5-Limit Pentad **8**:16



	45	48	60	72	80	90
H –	C	Db	F	Ab	\Bb	C
M –	C	B	G	E	D	C

48

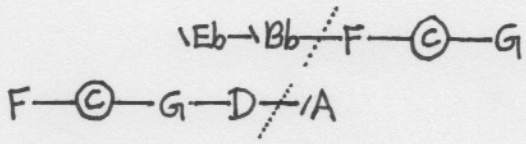
An important 3-Limit monochord, it establishes the 3-Limit Pentad in its smallest numbers. It is the monochord (Lüs) of Ling Lun in ancient China. Also, it forms Plato's Circle of the Essence (or existence) in the Timaeus construction. The dual is **54**:108



	48	54	64	72	81	96
H –	C	D	F	G	/A	C
M –	C	\Bb	G	F	\Eb	C

54

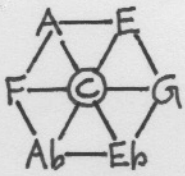
As the 3-Limit, it is the dual of Pentad **48**:96



54	64	72	81	96	108	
H–C	\Eb	F	G	\Bb	C	
M–C	/A	G	F	D	C	

60

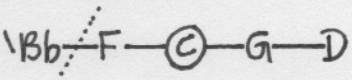
It is the most harmonius expression of the 5-Limit Chromatic Heptad, self-dualing, and symmetrical about the Tonic. It puts all of the consonances (both Perfect and Imperfect) in relation to the generator. The standard division of Ptolemy. It is implied for planetary attribution by Martianus, Pliny, Censorinus, Theon of Smyrna, and Achilles Tatios



60	72	75	80	90	96	100	120
H–C	Eb	E	F	G	Ab	A	C
M–C	A	Ab	G	F	E	Eb	C

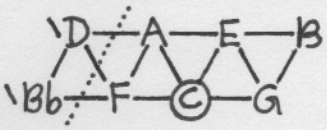
72

The most eminant of the planetary monochords, implied by many ancient authors. As the 3-Limit, it is the most harmonious position of the Pentad, symmetrical about the Tonic and self-dualing

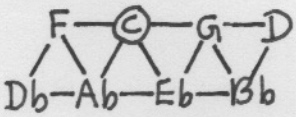


72	81	96	108	128	144
H–C	D	F	G	\Bb	C
M–C	\Bb	G	F	D	C

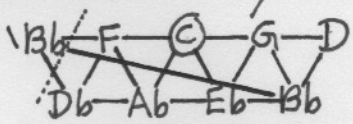
As the 5-Limit, it establishes the "full" Diatonic Ogdoad in smallest numbers. It's dual is **120**:240

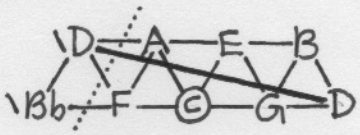


72	80	90	96	108	120	128	135	144
H–C	\D	E	F	G	A	\Bb	B	C
M–C	Bb	Ab	G	F	Eb	D	Db	C

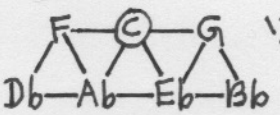


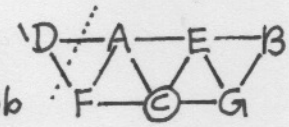
If the element 81 (H–D, M–\Bb) is added to the scale, the result is the Ennead which juxtaposes the comma, thus making the comma boundary explicit





As the 5-Limit, it is also the dual of **30**:60

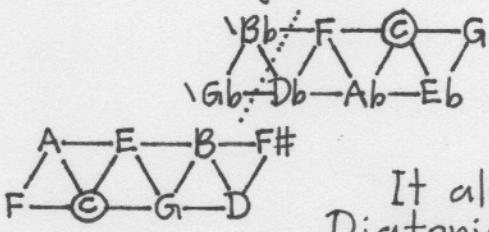




72	80	90	96	108	120	135	144
H–C	\D	E	F	G	A	B	C
M–C	Bb	Ab	G	F	Eb	Db	C

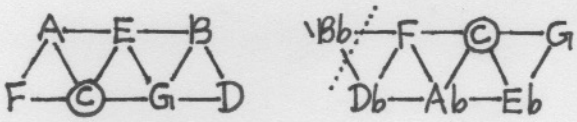
90

As the 5-Limit, it forms a particularly harmonious Diatonic Ogdoad, whose dual is **96**:192



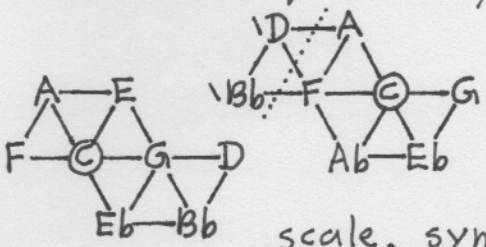
90	96	108	120	128	135	144	160	180
H – C	Db	Eb	F	\Gb	G	Ab	\Bb	C
M – C	B	A	G	F#	F	E	D	C

It also forms the dual of the 5-Limit Diatonic Heptad **24**:48, putting the 'natural' major scale on the monochord



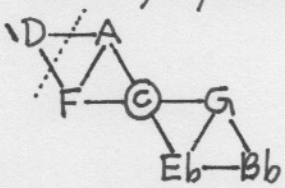
90	96	108	120	135	144	160	180	
H – C	Db	Eb	F	G	Ab	\Bb	C	
M – C	B	A	G	F	E	D	C	

As the 5-Limit, it forms the scale given by Achilles Tatios for planetary attribution. Dual is **120**:240



90	100	108	120	135	144	150	160	180
H – C	\D	Eb	F	G	Ab	A	\Bb	C
M – C	Bb	A	G	F	E	Eb	D	C

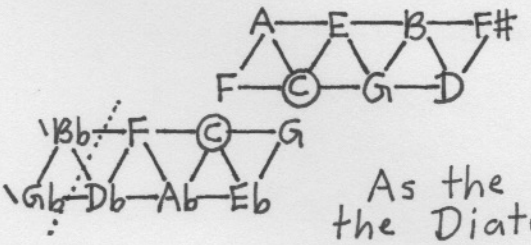
It also forms a prominant Greek scale, symmetrical about the Tonic, self-dual



90	100	108	120	135	150	160	180	
H – C	\D	Eb	F	G	A	Bb	C	
M – C	Bb	A	G	F	Eb	\D	C	

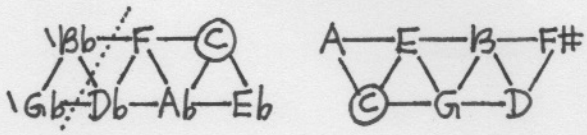
96

As the 5-Limit, it forms a very harmonious position of the Diatonic Ogdoad, the dual of the Diatonic Ogdoad **90**:180. Hinted at by Plato (Timaeus)



96	108	120	128	135	144	160	180	192
H – C	D	E	F	F#	G	A	B	C
M – C	\Bb	Ab	G	\Gb	F	Eb	Db	C

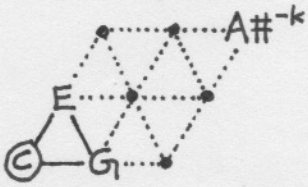
As the 5-Limit, it forms the dual of the Diatonic Heptad **45**:90

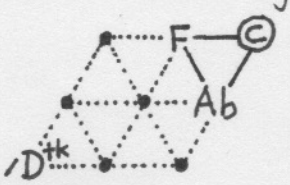


96	108	120	135	144	160	180	192	
H – C	D	E	F#	G	A	B	C	
M – C	\Bb	Ab	\Gb	F	Eb	Db	C	

105

As the 7-Limit, it is the dual of the 7-Limit Tetrad **4**: 8, (the Elemental Cosmological Monochord)

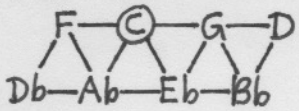




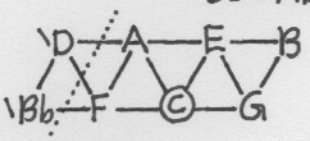
105	:120	:140	:168	:210
H – C	D+k	F	Ab	C
M – C	A#-k	G	E	C

120

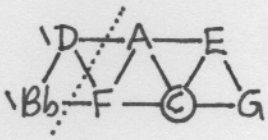
As the 5-Limit, it forms the Diatonic Ogdoad which is the dual of the Cosmological Diatonic Ogdoad **72**: 144

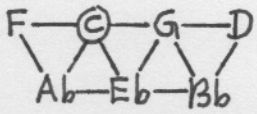


120	:128	:135	:144	:160	:180	:192	:216	:240
H – C	Db	D	Eb	F	G	Ab	Bb	C
M – C	B	\Bb	A	G	F	E	\D	C



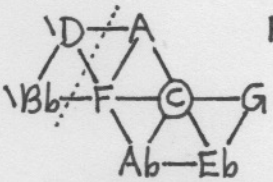
As the 5-Limit, it is the dual of the Cosmological Heptad **36**: 72

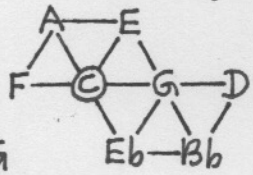




120	:135	:144	:160	:180	:192	:216	:240
H – C	D	Eb	F	G	Ab	Bb	C
M – C	\Bb	A	G	F	E	\D	C

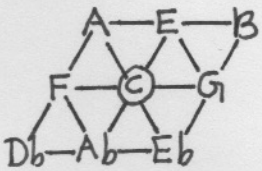
As the 5-Limit, it forms the scale which is the dual of **90**: 180, the scale given by Achilles Tatios





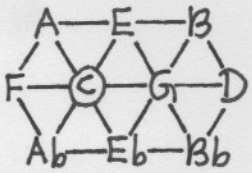
120	:135	:144	:150	:160	:180	:200	:216	:240
H – C	D	Eb	E	F	G	A	Bb	C
M – C	\Bb	A	Ab	G	F	Eb	\D	C

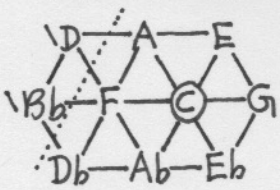
Also, it forms an important 5-Limit Chromatic Ennead, self-dualing



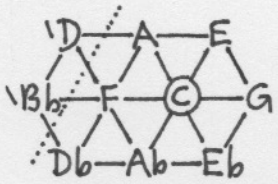
120	:128	:144	:150	:160	:180	:192	:200	:225	:240
H – C	Db	Eb	E	F	G	Ab	A	B	C
M – C	B	A	Ab	G	F	E	Eb	Db	C

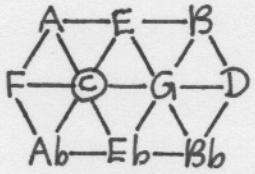
Also, as the 5-Limit, it forms an important Chromatic expansion of the Chromatic Heptad 60: 120, making the Chromatic Decad whose dual is **180**: 360





120	:135	:144	:150	:160	:180	:192	:200	:216	:225	:240
H – C	D	Eb	E	F	G	Ab	A	Bb	B	C
M – C	\Bb	A	Ab	G	F	E	Eb	\D	Db	C



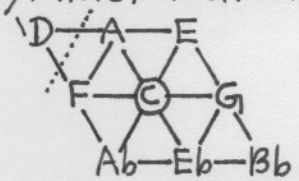


180

As the 5-Limit, it is the dual of the Chromatic decad **120**:240. This harmony is implied by the chromatic planetary monochords of Pliny, Theon, Martianus, Censorinus, and Achilles

180:	192 :	200 :	216 :	225 :	240 :	270 :	288 :	300 :	320 :	360
H – C	Db	\D	Eb	E	F	G	Ab	A	\Bb	C
M – C	B	Bb	A	Ab	G	F	E	Eb	D	C

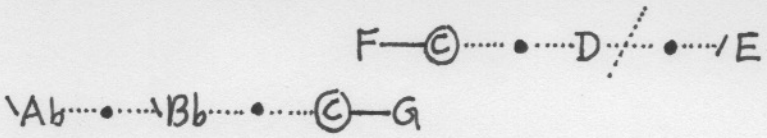
It is also the source for a core chromatic ennead which is symmetrical about the Tonic and self-dualing



180:	200 :	216 :	225 :	240 :	270 :	288 :	300 :	324 :	360
H – C	\D	Eb	E	F	G	Ab	A	Bb	C
M – C	Bb	A	Ab	G	F	E	Eb	\D	C

192

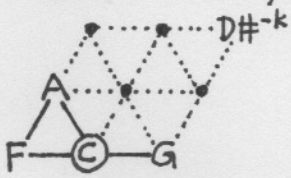
As the 3-Limit, it places the 'Pythagorean' Diatonic Tetrachord in the Mese position. This tetrachord is given by Plato (Timaeus), Nicomachus, Boethius, Ptolemy, Euclid, and others. This tetrachord is given in smallest numbers in the 3-Limit division **128**:256 (not presented in this set). Its dual is 3-Limit **162**:324 (not presented)

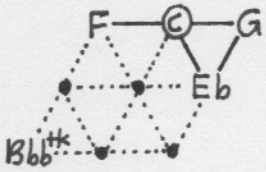


192:	216 :	243 :	256 :	(384)
H – C	D	/E	F	(C)
M – C	\Bb	\Ab	G	(C)

210

As the 7-Limit, it is the dual of pentad **6**:12

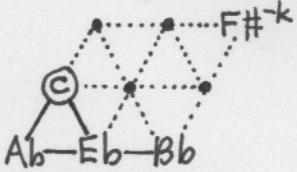


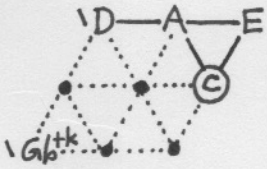


210:	252 :	280 :	315 :	360 :	420
H – C	Eb	F	G	Bbb+k	C
M – C	A	G	F	D#-k	C

252

As the 7-Limit, it is the dual of pentad **5**:10
It is obliquely refered to by Plato in Laws where the number of 'citizens' in the city of Magnesia is 5040, referring to 7-Limit monochord **2520**:5040.



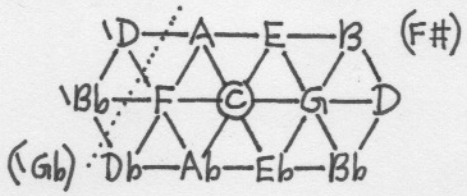


252:	280 :	315 :	360 :	420 :	504
H – C	\D	E	\Gb+k	A	C
M – C	Bb	Ab	F#-k	Eb	C

360

Important 5-Limit division forming the essential chromatic scale, associated with the calendar and measurement of time (the year number) and space (degrees of the circle). Structurally, it is symmetrical about the Tonic and self-dualing; unless one adds the additional element shown in brackets. Then the monochord and harmonic series become differentiated with dual **480**: 960 (not presented here)

360	384	400	405	432	450	480	(512)	540	576	600	640	648	675	720
H – C	Db	\D	D	Eb	E	F	(\Gb)	G	Ab	A	\Bb	Bb	B	C
M – C	B	Bb	\Bb	A	Ab	G	(F#)	F	E	Eb	D	\D	Db	C



384

As the 3-Limit, it gives the Diatonic Heptad in its smallest numbers. As such, it underlies the planetary monochord of Nicomachus. Dual is **486**: 972

F — C — G — D — /A — /E — /B+

\Db- — \Ab — \Eb — \Bb — F — C — G

384	432	486	512	576	648	729	768
H – C	D	/E	F	G	/A	/B+	C
M – C	\Bb	\Ab	G	F	\Eb	\Db-	C

432

As the 3-Limit, it is an important mode of the Diatonic Heptad, symmetrical about the tonic and self-dualing.

\Eb — \Bb — F — C — G — D — /A

432	486	512	576	648	729	768	864
H – C	D	\Eb	F	G	/A	\Bb	C
M – C	\Bb	/A	G	F	\Eb	D	C

486

As the 3-Limit, it is the dual of **384**: 768

\Db- — \Ab — \Eb — \Bb — F — C — G

F — C — G — D — /A — /E — /B+

486	512	576	648	729	768	864	972
H – C	\Db-	\Eb	F	G	\Ab	\Bb	C
M – C	/B+	/A	G	F	/E	D	C

576

As the 3-Limit, an important mode of the Diatonic Heptad mentioned in planetary monochords by Plato (myth of Er), Nicomachus, Dio Cassius, and Al Kindi. The dual is **648**: 1296

\Bb — F — C — G — D — /A — /E

\Ab — \Eb — \Bb — F — C — G — D

576	648	729	768	864	972	1024	1152
C	D	/E	F	G	/A	\Bb	C
C	\Bb	\Ab	G	F	\Eb	D	C

648

As the 3-Limit, it is the dual of the planetary Diatonic Heptad 576:1152

\Ab—\Eb—\Bb—F—(C)—G—D

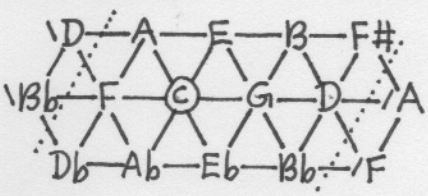
\Bb—F—(C)—G—D—/A—/E

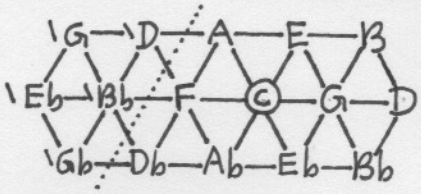
	648	729	768	864	972	1024	1152	1296
H –	C	D	\Eb	F	G	\Ab	\Bb	C
M –	C	\Bb	/A	G	F	/E	D	C

1080

As the 5-Limit, it defines a harmonious expansion of the triple-strand chromatic harmony 360:720. The dual is 1440:2880

	1080	1152	1200	1215	1280	1296	1350	1440	1536	1600	1620	1728	1800	1920	1944	2025	2160
H –	C	Db	\D	D	\Eb	Eb	E	F	\Gb	\G	G	Ab	A	\Bb	Bb	B	C
M –	C	B	Bb	\Bb	/A	A	Ab	G	F#	/F	F	E	Eb	D	\D	Db	C





1152

As the 3-Limit, it is the most prestigious of the Diatonic Ogdoads, presenting this harmony in smallest numbers. It is the 3-Limit counterpart to the Cosmological Ogdoad (72:144). It is the monochord of Euclid's Sectio Canonis and also associated with Plato (myth of Er), Nicomachus, Cicero (Boethius), Dio Cassius, Ptolemy, Eratosthenes, and Al Kindi. It is the 'full' diatonic attributed to Pythagoras. Dual is 1944:3888

	1152	1296	1458	1536	1728	1944	2048	2187	2304
H –	C	D	/E	F	G	/A	\Bb	/B+	C
M –	C	\Bb	\Ab	G	F	\Eb	D	\Db-	C

\Db—\Ab—\Eb—\Bb—F—(C)—G—D

\Bb—F—(C)—G—D—/A—/E—/B+

1296

An important mode of the 3-Limit Diatonic Ogdoad, referred to by Achilles Tatios. The dual is 1728:3456

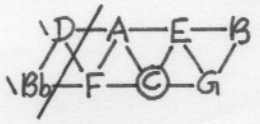
	1296	1458	1536	1728	1944	2048	2187	2304	2592
H –	C	D	\Eb	F	G	\Ab	/A	\Bb	C
M –	C	\Bb	/A	G	F	/E	\Eb	D	C

\Eb—\Bb—F—(C)—G—D—/A—/E

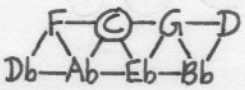
\Ab—\Eb—\Bb—F—(C)—G—D—/A

Figure 8: The Planetary Monochord (Celestial Spheres)

symbolizing the 'sacred' 5-Limit Ogdoad in smallest terms



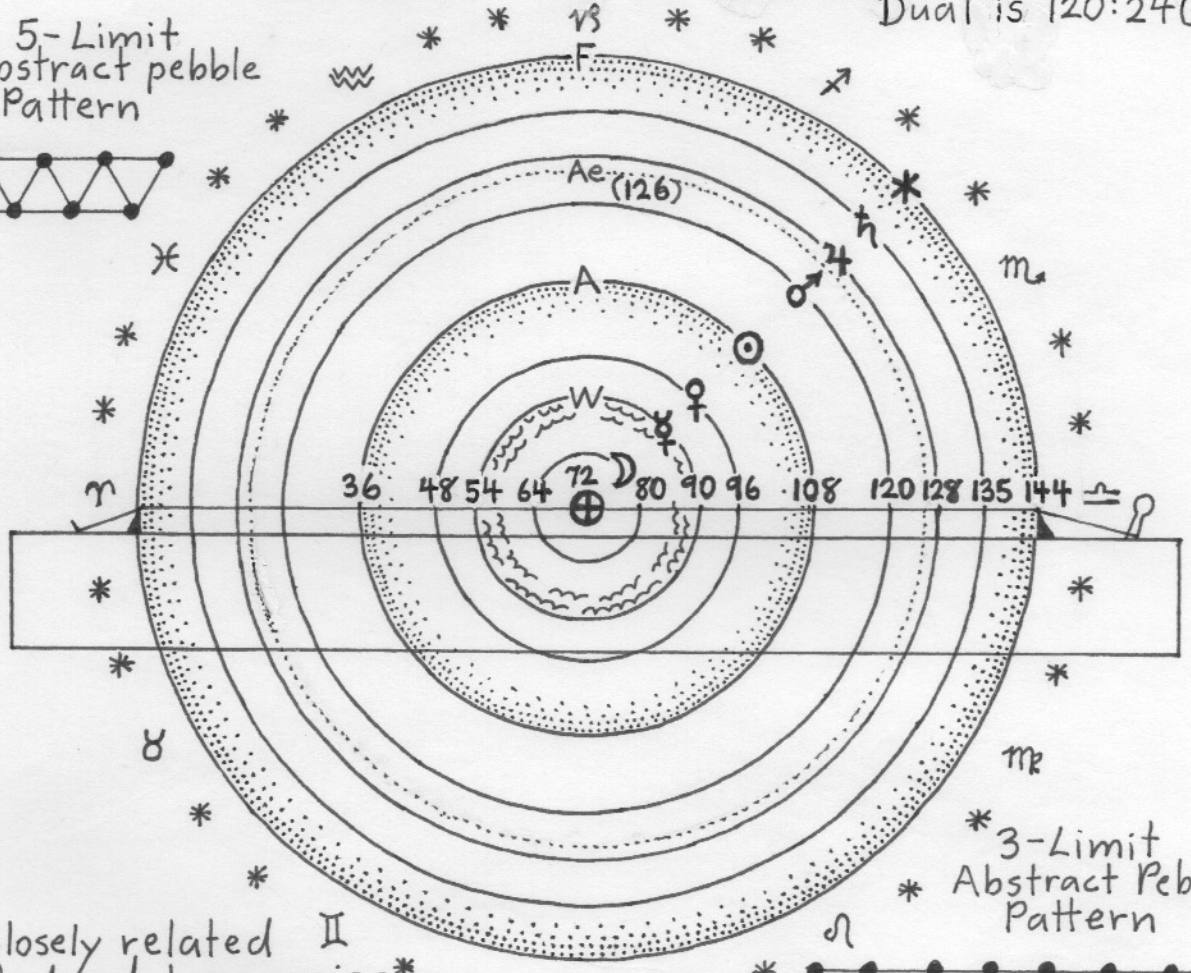
72:144



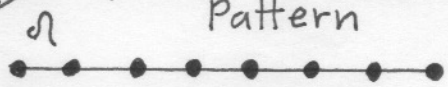
⊕	☽	☿	♀	☉	♂	♃	♄	✱
72 :	80 :	90 :	96 :	108 :	120 :	128 :	135 :	144
C	\D	E	F	G	A	\Bb	B	C
C	Bb	Ab	G	F	Eb	D	Db	C

Dual is 120:240

5-Limit Abstract pebble Pattern



3-Limit Abstract Pebble Pattern



Closely related Ogdoad harmonies

	⊕	☽	☿	♀	☉	♂	♃	♄	✱
Euclid (Archytas)	**1152** :	1296 :	1458 :	1536 :	1728 :	1944 :	2048 :	2187 :	2304
3-Limit Ogdoad	C	D	/E	F	G	/A	\Bb	/B+1	C
in smallest terms	C	\Bb	\Ab	G	F	\Eb	D	\Db-1	C

Plato's Timaeus	**1536** :	1728 :	1944 :	2048 :	2187 :	2304 :	2592 :	2916 :	3072
3-Limit Ogdoad	C	D	/E	F	/F#+1	G	/A	/B+1	C
"World Soul"	C	\Bb	\Ab	G	F#-1	F	\Eb	\Db-1	C

Plato's Timaeus	**96** :	108 :	120 :	128 :	135 :	144 :	160 :	180 :	192
5-Limit Ogdoad	C	D	E	F	F#	G	A	B	C
	C	\Bb	Ab	G	\Gb	F	Eb	Db	C

Figure 11: Key to the "pebble arithmetic" patterns

2-Limit Cyclical identity ⟶ Octave Invariance ©

3-Limit A line of musical fifths and fourths

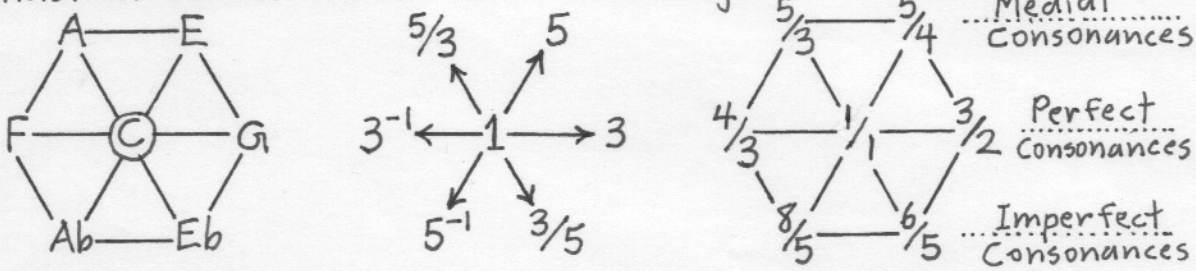
$3^{-n} \longleftarrow 3^{-3}\ 3^{-2}\ 3^{-1}\ 1\ \ 3\ \ 3^{2}\ \ 3^{3} \longrightarrow 3^{n}$

$2^5/3^3\ \ 2^4/3^2\ \ 2^2/3\ \ 1/1\ \ 3/2\ \ 3^2/2^3\ \ 3^3/2^4$ "floating" octaves

Eb—Bb—F—©—G—D—A Pitch notation

5-Limit Opens the line onto a 'surface' consisting of an open ended Triad Lattice - formally called the '5-Limit Tri-axial Matrix

The musical consonances form a hexagon around the One

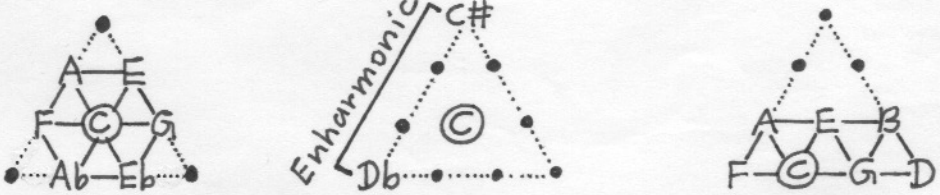


Every type of Harmony has its own pattern signature

Major Triad | Minor Triad | Diatonic Scales (5-Limit Heptads) | Chromatic Scale

The musical consonances within the Tetractys

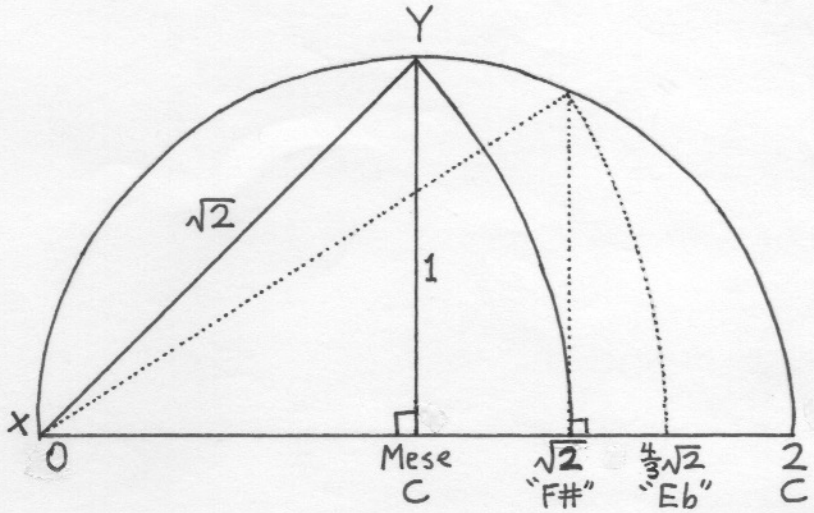
The enharmonic and the diatonic scale within Tetractys



Enharmonic
Chromatic
Diatonic

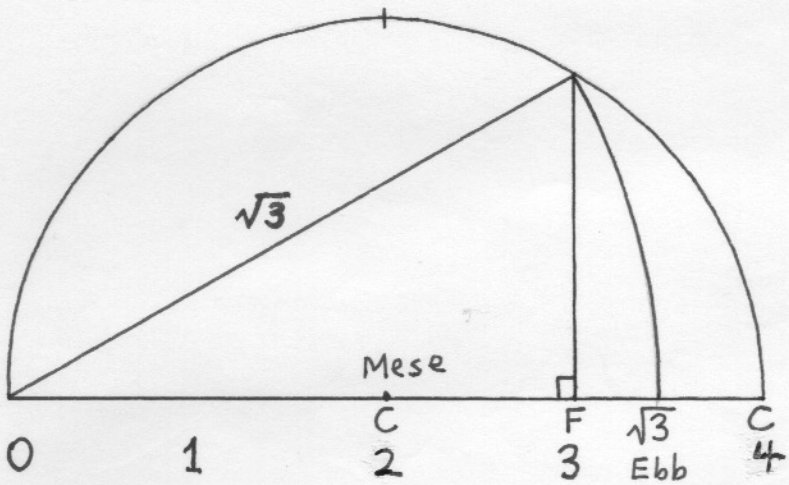
The three 'levels' house the genera, as shown above

Figure 15: The Geometrical Proportion on the Monochord

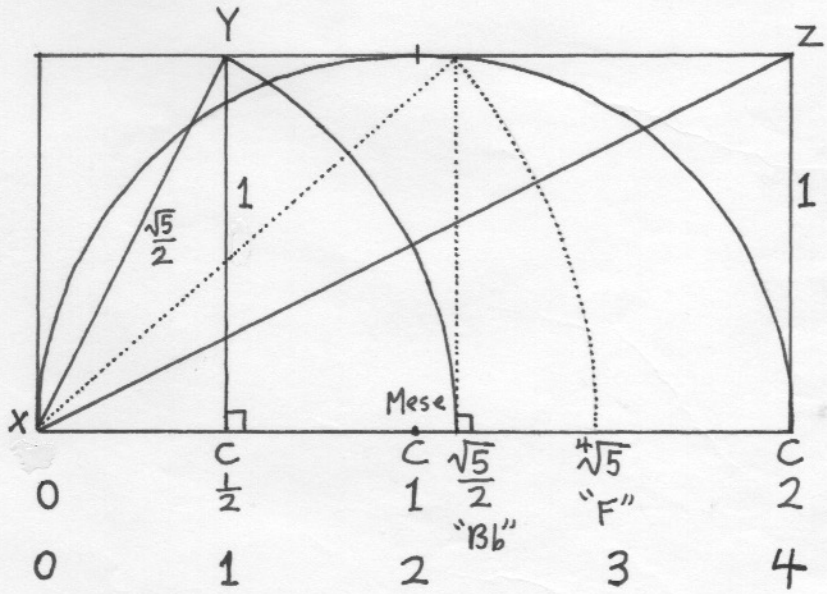


Use Monochord Series division 1:2

In the first construction, a perpendicular (Gnomon) is erected at the Mese to intersect the 'dome' at Y. An arc is drawn with radius XY, which meets the string at the desired fret-mark. This point can then be used to erect another gnomon and find more irrational ratios.



In this case, a gnomon erected at F (using the division 2:3:4) yields the irrational ratio which is the mean between F-C. This point is used to find more irrationals from 53-E.T.



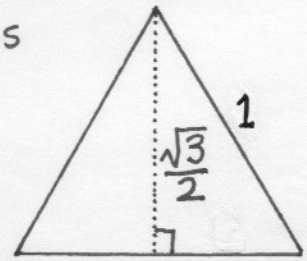
In this case, the semi-circle is enclosed within the 1:2 rectangle, whose diagonal (XZ) is $\sqrt{5}$. At the second octave point, erect a gnomon, and extend the arc to yield the Meantone ratio $\sqrt{5}/2$. In a second procedure, erect another gnomon to yield $\sqrt[4]{5}$, which is the meantone fifth (flat by one-fourth comma)

Figure 16: The Pythagorean association between the Sacred Roots (Elements) and Solid Geometry

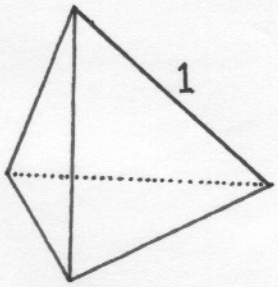
1:1 The Monad - Fire, heaven

V= number of vertices
F= number of faces
E= number of edges

V+F= E+2 (Euler)



Two-dimensional componant

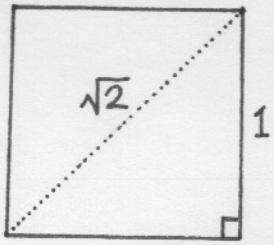


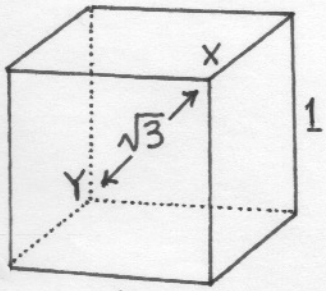
Tetrahedron
V=4 F=4 E=6
Self-dual

---

[1] 1:1 ⟶ **1:2*** The Dyad - Earth

Hexahedron (Cube)
V=8 F=6 E=12
Dual - Octahedron

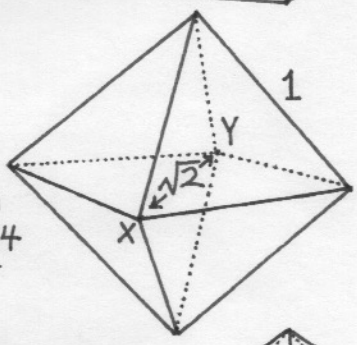




[2] 1:2 ⟶ **2:3:4*** The Triad - Air

Octahedron
V=6 F=8 E=12
Dual - Hexahedron

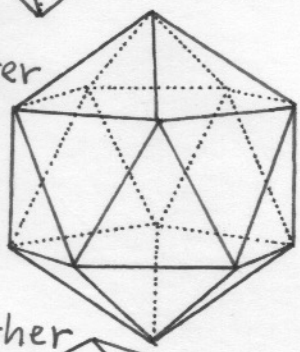
[Union of Hexa. + Octa.
→ Cuboctahedron
V=12, F=14, E=24



[3] 2:3:4 ⟶ **3:4:5:6*** The Pentad - Water

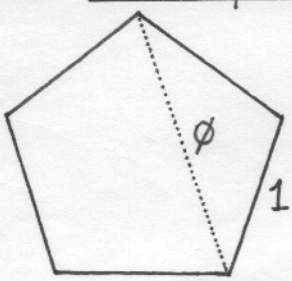
Icosahedron
V=12 F=20 E=30
Dual - Dodecahedron

[Union of Icosa. + Dodeca.
→ Icosadodecahedron
V=30, F=32, E=60



[4] 3:4:5:6 ⟶ **4:5:6:7:8*** The Heptad - Aether

Dodecahedron
V=20 F=12 E=30
Dual - Icosahedron



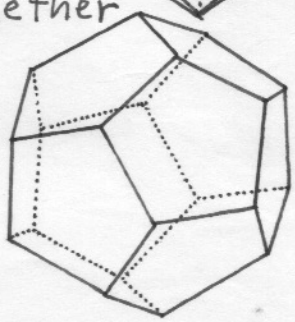
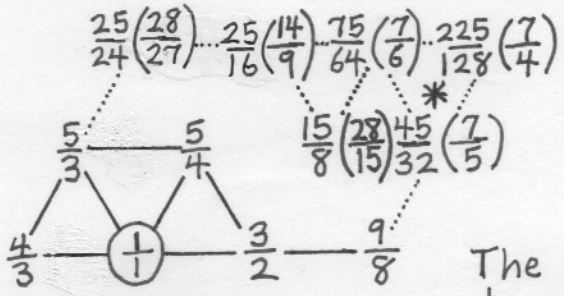
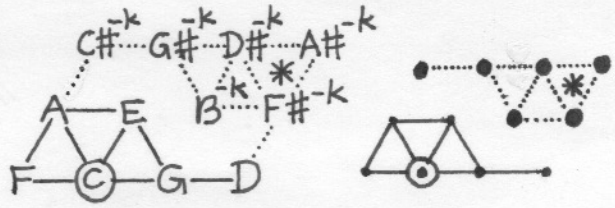


Figure 17: The 7-Limit Archetypes within the pebble arithmetic

A# (5-Limit) = 498 s A# (7-Limit) = 494 s klisma (k) = 4 s





The 7-Limit substitutions centre around two triads, marked by *, which are either k-raised or k-lowered from the 5-Limit

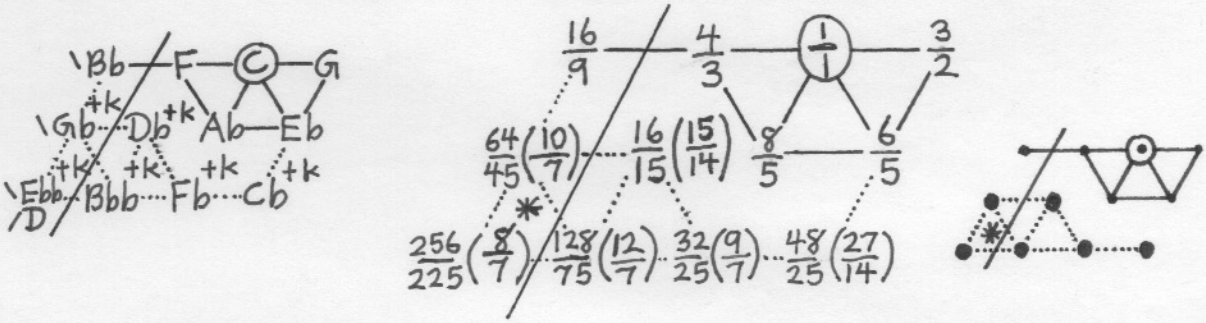


Figure 18: The schisma as "atomic unit" of Just Harmony

S	Ratio	Name	c	e	Se	Construction from s, k, ic, sc
0	1:1	Unison	0	0	0	
1	883:884	Schisma				s
4	224:225	klisma				k
10	2025:2048	Diaschisma C.				ic
11	80:81	Syntonic Comma	1			sc
12	73:74	Ditonic Comma	1			dc = sc + s
14	63:64	7-Lim Comma				ic + k
15	3072:3125	5-Limit				sc + k
21	125:128	Enharmonic (5-L)	2	1		sc + ic
25	35:36	Enharmonic (7-L)	2	1		sc + ic + k
32	27:28	7-L. Chromatic Se.	3	2		sc + ic + sc
36	24:25	Chromatic Se. (5-L)	3	2		sc + ic + sc + k
43	20:21	7-Lim. Medial Se.				sc + sc + sc + ic
46	243:256	3-Lim. Medial Se.	4		1	sc + sc + ic + ic + k
47	128:135	5-Lim Medial Se.	4		1	sc + sc + sc + ic + k
57	15:16	5-Lim Diatonic Se.	5	3	1	sc + sc + sc + ic + ic + k
61	14:15	7-Lim. Diatonic Se.		3		sc + ic + ic + ic + ic + ic
68	25:27	5-Lim. "acute" Se.	6			sc + sc + sc + sc + ic + ic + k
77	11:12	"3/4th tone"	7	4		sc + sc + sc + ic + ic + ic + ic + k
↓						

The primary "building blocks" of Just Harmony

Figure 19: The Circular Graph shows metrical properties of musical intervals. Here it is resolved into commas.

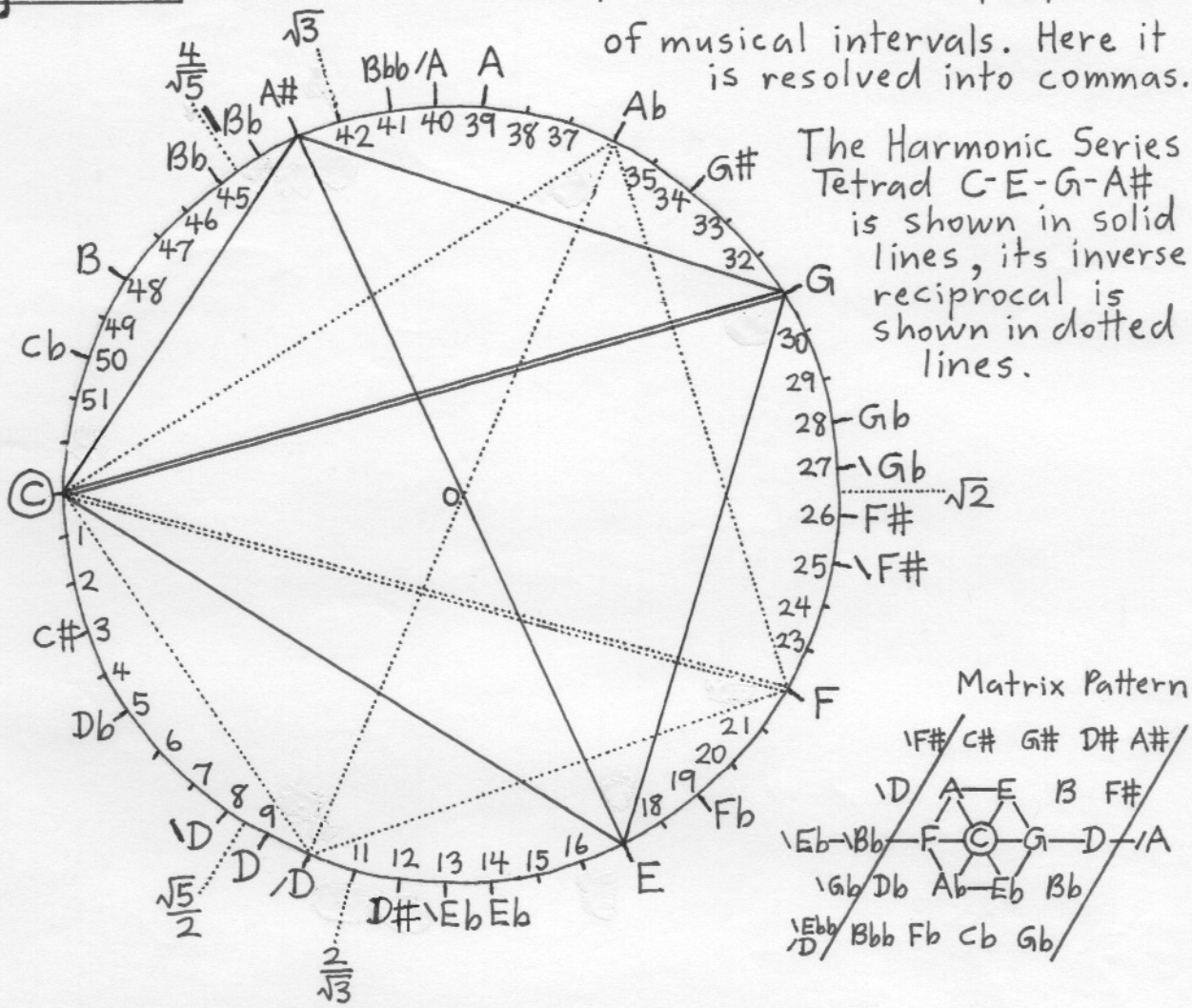


Figure 20: The Greater Perfect System (Also, see **Figure 32**)

The first monochord division to place the Tetrachord Frames

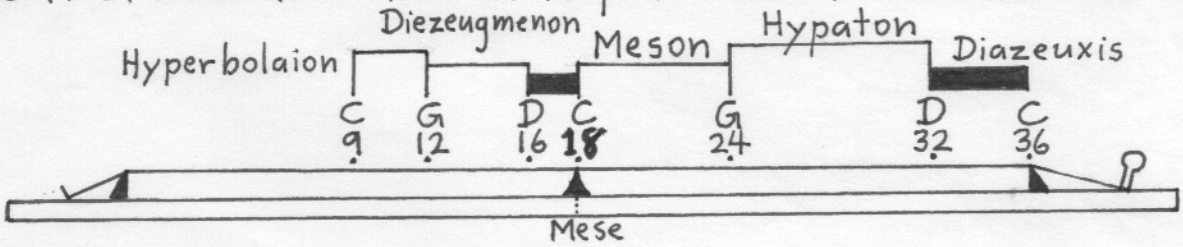


Figure 21: How to find the 5th Harmonic position on the monochord (This is not a classical procedure).

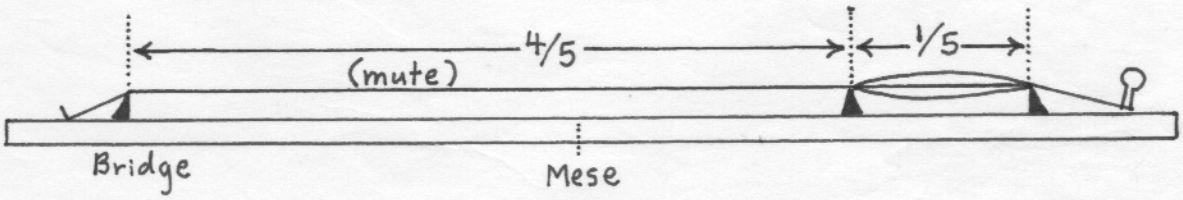
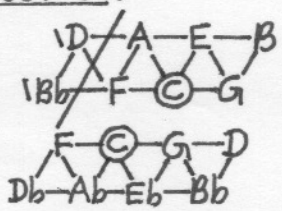
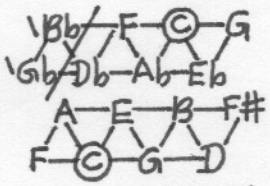


Figure 22: The 5-Limit and 3-Limit Diatonic Ogdoads
The four most harmonius modes:

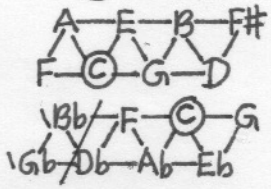
72:144	**72**:	80:	90:	96:	108:	120:	128:	135:	144
Dual	C	\D	E	F	G	A	\Bb	B	C
120:240	C	Bb	Ab	G	F	Eb	D	Db	C



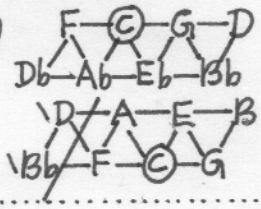
90:180	**90**:	96:	108:	120:	128:	135:	144:	160:	180
Dual	C	Db	Eb	F	\Gb	G	Ab	\Bb	C
96:192	C	B	A	G	F#	F	E	D	C



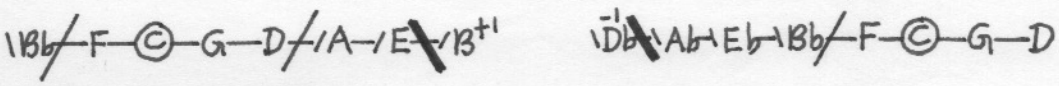
96:192	**96**:	108:	120:	128:	135:	144:	160:	180:	192
Dual	C	D	E	F	F#	G	A	B	C
90:180	C	\Bb	Ab	G	\Gb	F	Eb	Db	C



120:240	**120**:	128:	135:	144:	160:	180:	192:	216:	240
Dual	C	Db	D	Eb	F	G	Ab	Bb	C
72:144	C	B	\Bb	A	G	F	E	\D	C

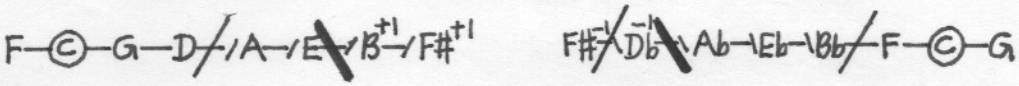


1152:2304	**1152**:	1296:	1458:	1536:	1728:	1944:	2048:	2187:	2304
Dual	C	D	/E	F	G	/A	\Bb	/B^{+1}	C
1944:3888	C	\Bb	\Ab	G	F	\Eb	D	\Db^{-1}	C



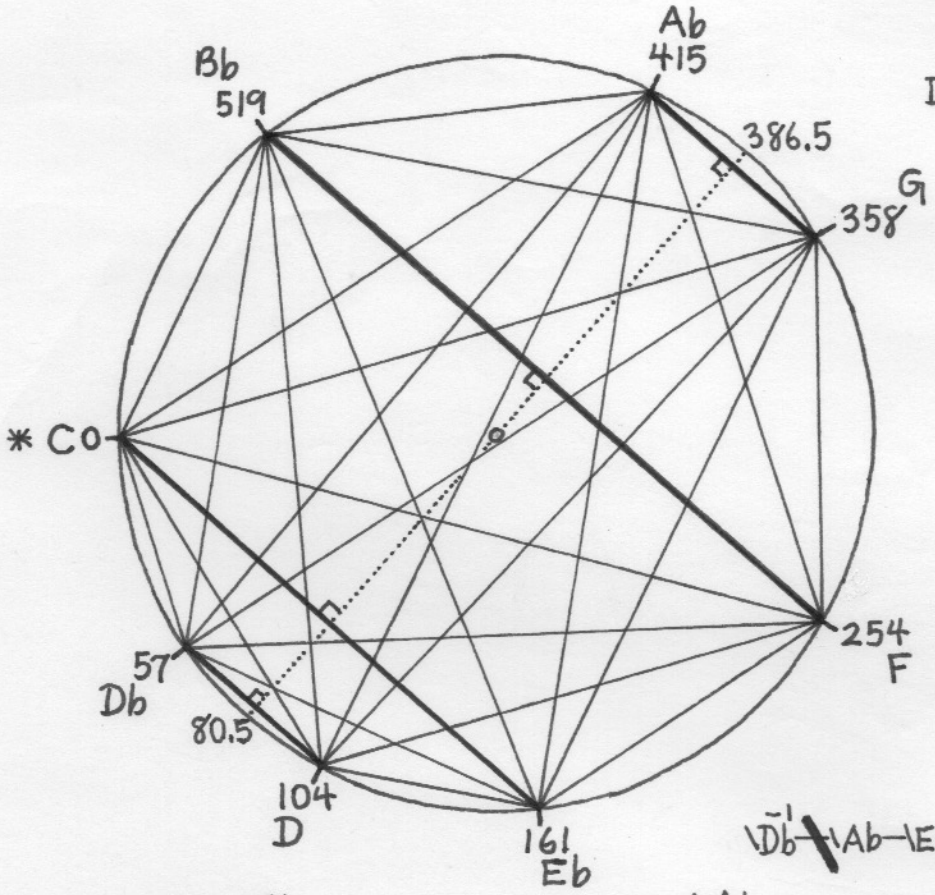
1458:2916	**1458**:	1536:	1728:	1944:	2048:	2187:	2304:	2592:	2916
Dual	C	\Db^{-1}	\Eb	F	F#$^{-1}$	G	\Ab	\Bb	C
1536:3072	C	/B^{+1}	/A	G	/F#$^{+1}$	F	/E	D	C

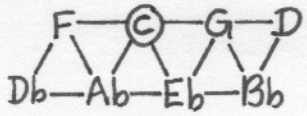
1536:3072	**1536**:	1728:	1944:	2048:	2187:	2304:	2592:	2916:	3072
Dual	C	D	/E	F	/F#$^{+1}$	G	/A	/B^{+1}	C
1458:2916	C	\Bb	\Ab	G	F#$^{-1}$	F	\Eb	\Db^{-1}	C



1944:3888	**1944**:	2048:	2187:	2304:	2592:	2916:	3072:	3456:	3888
Dual	C	\Db^{-1}	D	\Eb	F	G	\Ab	\Bb	C
1152:2304	C	/B^{+1}	\Bb	/A	G	F	/E	D	C

Figure 23: The Ogdoad Monochords 72:144 & 1152:2304
on the Circular Graph – schisma resolution

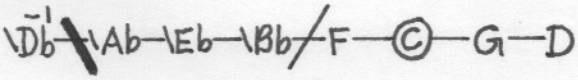


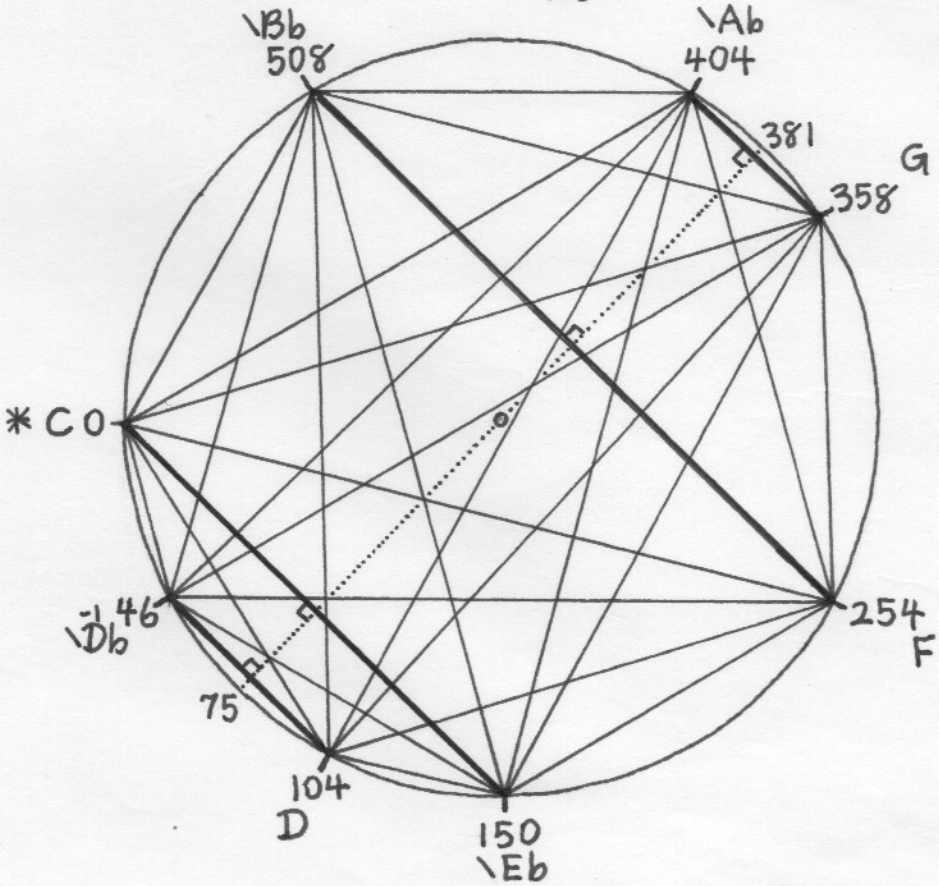


Symmetry

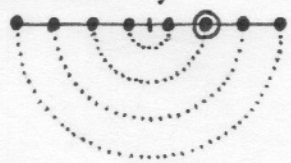
386.5 − 80.5 = 306s
306s = $\sqrt{2}$

Intervals which are perpendicular to the Axis are emphasized on both Harmonies.





Symmetry



381 − 75 = 306s
306s = $\sqrt{2}$

Interval Triangles:

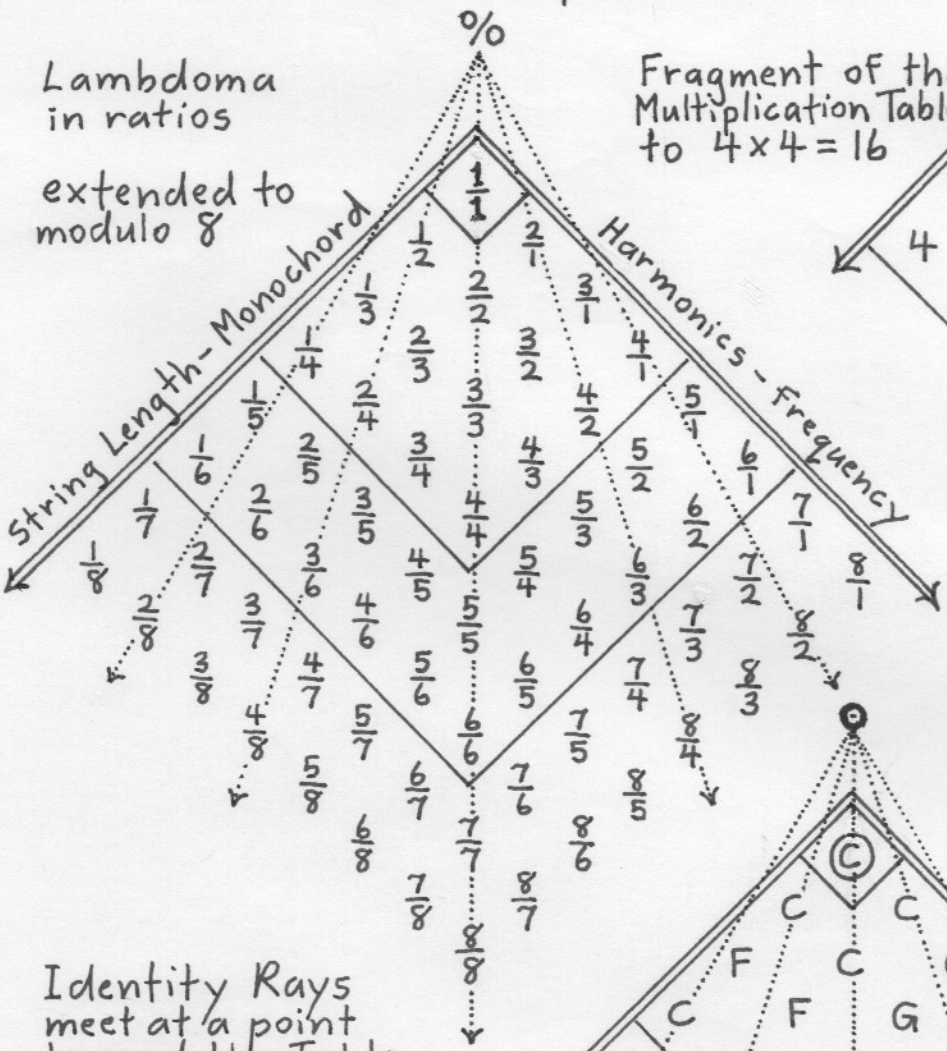
161
104 104
57 47 57
C Db D Eb

150
104 104
46 58 46
C \Db⁻¹ D \Eb

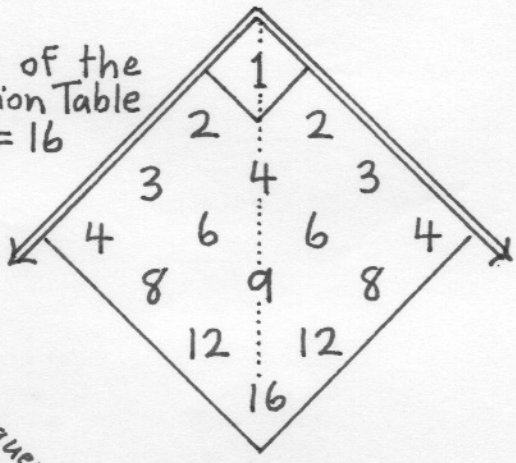
Figure 24: An alternative source for the musical ratios
The Multiplication Table → Lambdoma

Lambdoma in ratios

extended to modulo 8

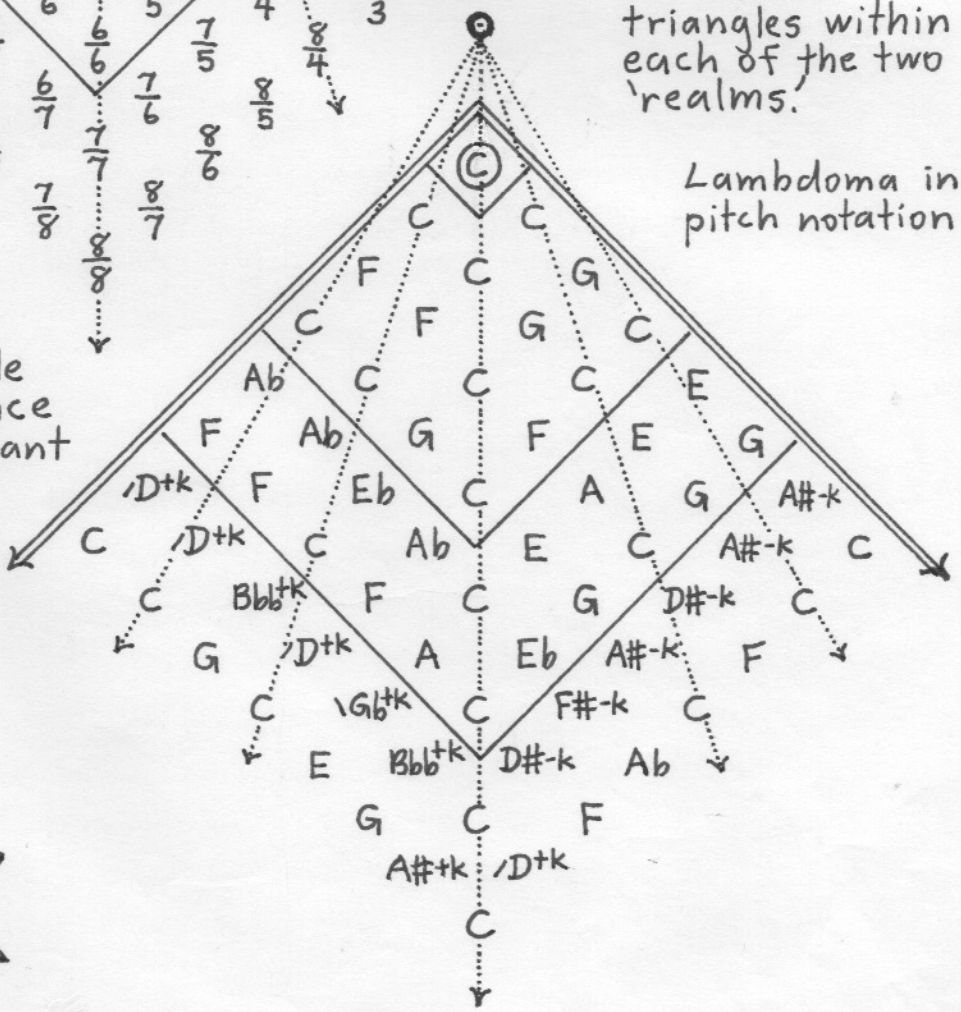


Fragment of the Multiplication Table to 4×4 = 16



The Lambdoma is based on a square (90°) grid, yet it manifests the interval triangles within each of the two 'realms.'

Lambdoma in pitch notation



Identity Rays meet at a point beyond the Table implying Silence or the transcendant Source of the manifesting emanation

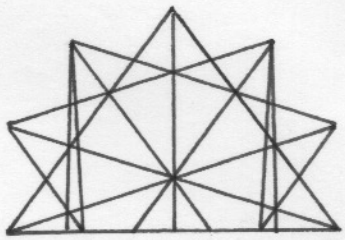
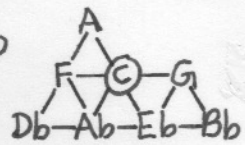


Figure 25: The Old-Babylonian Reciprocal Table with god-names interpreted as the Monochord Series 30:60

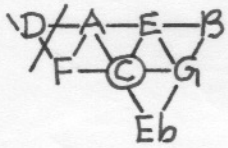
Table:

Base 10		Base 60		Pitch o	Pitch u	Ratio	god-name	
60	1	60	1	Db	B	1:60 = 1:60		
30	2	30	2	Db	B	2:60 = 1:30		
20	3	20	3	Ab	E	3:60 = 1:20		
15	4	15	4	Db	B	4:60 = 1:15		
12	5	12	5	F	G	5:60 = 1:12		
10	6	10	6	Ab	E	6:60 = 1:10	Adad	
7.5	8	7;30	8	Db	B	8:60 = 2:15		
6.6666...	9	6;40	9	Eb	A	9:60 = 3:20		
6	10	6	10	F	G	10:60 = 1:6	Bel-Marduk	
5	12	5	12	Ab	E	12:60 = 1:5	Nergal	
4	15	4	15	C	C	15:60 = 1:4	Ishtar	
3.75	16	3;45	16	Db	B	16:60 = 4:15		
3.3333...	18	3;20	18	Eb	A	18:60 = 3:10		
3	20	3	20	F	G	20:60 = 1:3	Shamash	
2.5	24	2;30	24	Ab	E	24:60 = 2:5		
2.4	25	2;24	25	A	Eb	25:60 = 5:12		
2.2222...	27	2;13,20	27	Bb	\D	27:60 = 9:20		♇
2	30	2	30	C	C	**30**:60 = 1:2	Sin	⊕
1.875	32	1;52,30	32	Db	B	32:60 = 8:15		☽
1.6666...	36	1;40	36	Eb	A	36:60 = 3:5		☿
1.5	40	1;30	40	F	G	40:60 = 2:3	Ea-Enki	♀
1.3333...	45	1;20	45	G	F	45:60 = 3:4		☉
1.25	48	1;15	48	Ab	E	48:60 = 4:5		♂
1.2	50	1;12	50	A	Eb	50:60 = 5:6	(Bel)-Enlil	♃
1.1111...	54	1;6,40	54	Bb	\D	54:60 = 9:10		♄
1	60	1	60	C	C	60:60 = 1:1	Anu-An	*
0.9375	64	0;56,15	1;4	Db	B	64:60 = 16:15	not deified	
0.8333...	72	0;50	1;12	Eb	A	72:60 = 6:5	not deified	
0.8	75	0;48	1;15	E	Ab	75:60 = 5:4	not deified	
0.75	80	0;45	1;20	F	G	80:60 = 4:3	not deified	
0.74074...	81	0;44,26,40	1;21	/F	\G	81:60 = 27:20	not deified	

Matrix pattern: o



u



Monochord:

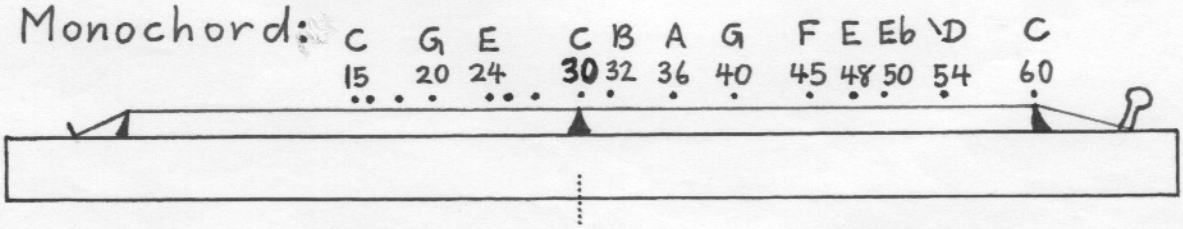


Figure 26: Some number contexts which have been confused with Canonics.

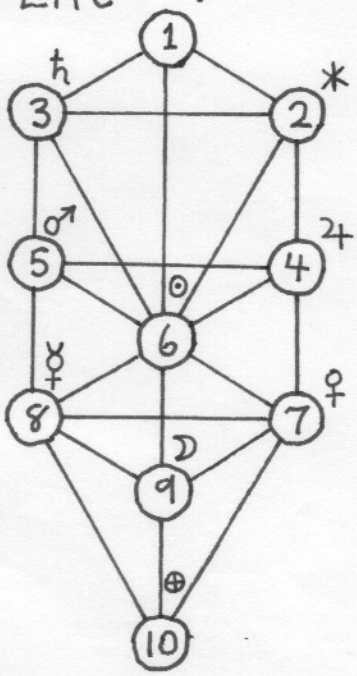
The Planetary Square of Saturn

4	9	2
3	5	7
8	1	6

Number of boxes in square = 3 × 3 = 9
Sum of any line: (horizontal, vertical, diagonal) 15
Sum of any symmetrical group of 4 numbers 20
Sum of perimeter values 40 (multiple of above)
Sum of entire square 45

The Planets on the Kabbalistic "Tree of Life"

Number	Planet	Roman	Greek	Metal
1	The One			
2	✳			
3	♄	Saturn	Kronos	Lead
4	♃	Jupiter	Zeus	Tin
5	♂	Mars	Ares	Iron
6	☉	Sun	Helios	Gold
7	♀	Venus	Aphrodite	Copper
8	☿	Mercury	Hermes	Mercury
9	☽	Luna	Selene	Silver
10	⊕	Terra	Gaia	All



An Astrological Chart

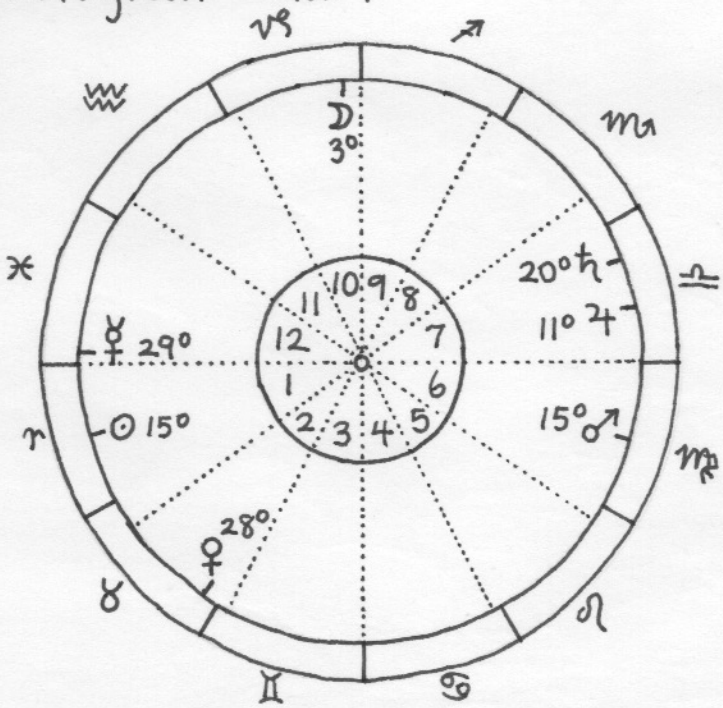


Figure 27: Symmetry values of various harmonies (all Diatonic)

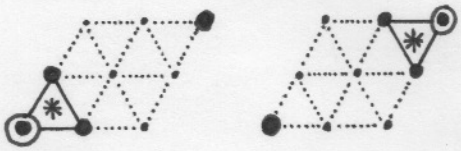
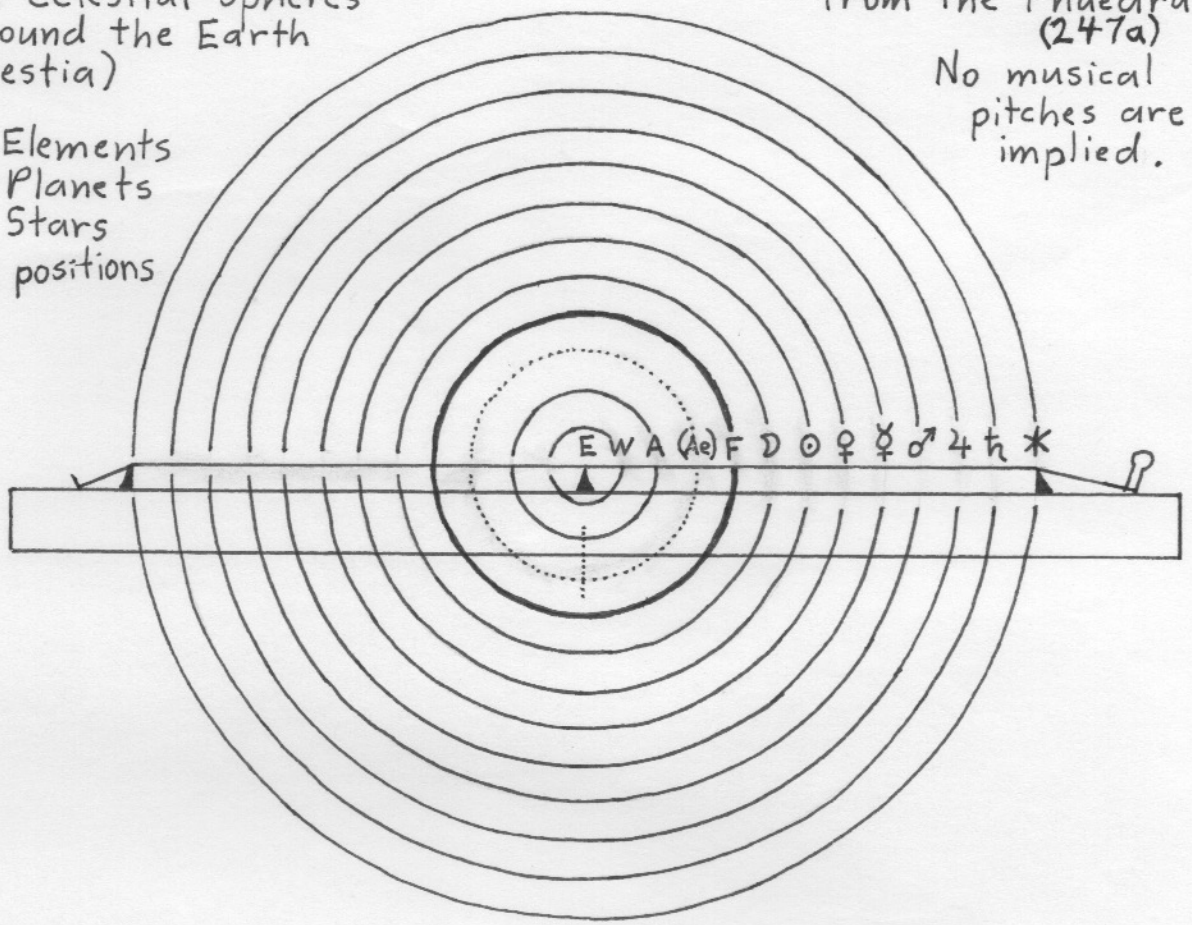
Matrix Pattern	Name	Monochord	Non-sym pair	Sym.
3-Limit				
	Triad	6:12		✓
	Tetrad	16:32		✓
	Pentad	48:96		✓
	Hexad	128:256		✓
	Heptad	384:768		✓
	Ogdoad	1152: 2304		✓
5-Limit				
	Triads	3:6 8:16	✓	
	Tetrads	8:16 8:16		✓
	Pentads	8:16 24:48	✓	
	Hexads	24:48 24:48		✓
	Heptads	24:48 72:144	✓	
	Ogdoads	72:144 72:144		✓
The 7-Limit Tetrad is an incipiant chromatic pattern				
		4:8 60:120	✓	

Figure 28: Two Platonic 'Re-interpretations' of the traditional Theory of the 'Music of the Spheres':

11 Celestial Spheres around the Earth (Hestia)

4 Elements
7 Planets
1 Stars
12 positions

- from the Phaedrus (247a)
No musical pitches are implied.



616c

- From the Myth of Er (The Republic) the eight whorls of the spindle of necessity are assigned to the octave divided into disjunct tetrachords:

Sphere	Name of Note	Tetrachord	Pitch: 3-Limit		Pitch: 5-Limit	
✳	Nete	diezeug-menon	G - 384	E	G - 24	E
♄	Paranete		F - 432	\D	F - 27	\D
♃	Trite		\Eb - 486	\C	Eb - 30	C
♂	Paramese		D - 512	B	D - 32	B
☿	Mese	Meson	C - **576**	A	C - **36**	A
♀	Lichanos		\Bb - 648	\G	Bb - 40	G
☉	Parhypate		\Ab - 729	\F	Ab - 45	F
☽	Hypate		G - 768	E	G - 48	E
⊕?	(Proslambanomenos)		(C) - (1152)	(A)	(C) - (72)	(A)

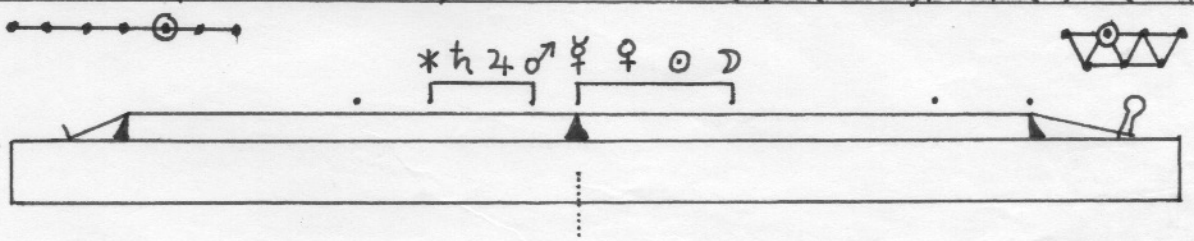
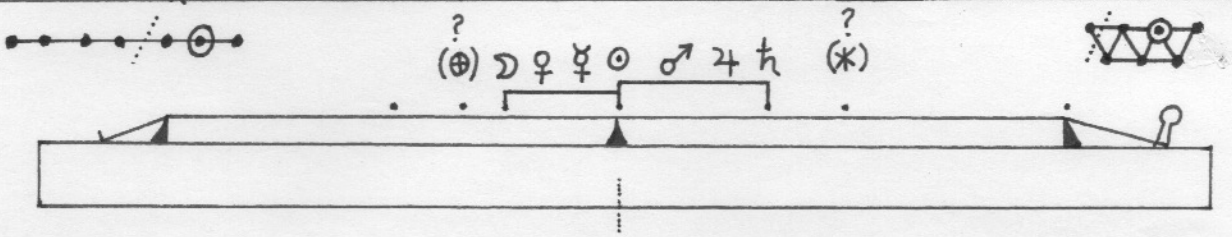


Figure 29: A Post Hellenistic Re-interpretation of the traditional 'Music of the Spheres'

Nicomachus' account from the Manual of Harmonics:

Planet	Name of Note	Tetrachord	Pitch: 3-Limit		Pitch: 5-Limit	
(Earth)?			(G) – (512)	E	(G) – (32)	(E)
☽	Nete		F – 576	\D	F – 36	\D
☿ (☿ ♀)	Paranete	Synemmenon	\Eb – 648	\C	Eb – 40	C
♀	Paramese (Trite)		\Db⁻¹ – 729	A#	Db – 45	\Bb
☉	Mese		C – **768**	A	C – **48**	A
♂	Lichanos (Hypermese)		\Bb – 864	\G	\Bb – 54	\G
♃	Parhypate	Meson	\Ab – 972	\F	Ab – 60	F
♄	Hypate		G – 1024	E	G – 64	E
(Stars)*			(F) – (1152)	(\D)	(F) – (72)	(\D)
	(Proslambanomenos)		(C) – (1536)	(A)	(C) – (96)	(A)

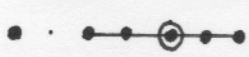


Nicomachus actually gives three versions; the two shown here (he reverses the placement of Mercury and Venus), and a third which reverses the scale direction, giving the falling order ♄ ♃ ♂ ☉ ☿ ♀ ☽. Note that in this reversed order, the Sun maintains its position on the Mese.

Roger Bragard (1929) re-interpreted Nicomachus to use disjunct rather than conjunct tetrachords, resulting in scale numbers like Plato's Myth of Er:

Planet	Name of Note	Tetrachord	Pitch: 3-Lim		Pitch: 5-Lim	
(Earth)?			(C)–(288)	(A)	(C)–(18)	(A)
☽	Nete	diezeug-	G – 384	E	G – 24	E
☿	Paranete	menon	F – 432	\D	F – 27	\D
♀	Paramese		D – 512	B	D – 32	B
☉	Mese		C – **576**	A	C – **36**	A
♂	Lichanos		\Bb – 648	\G	Bb – 40	G
♃	Parhypate	Meson	\Ab – 729	\F	Ab – 45	F
♄	Hypate		G – 768	E	G – 48	E
(*) ?			(D)–(1024)	(B)	(D)–(64)	(B)
	(Proslambanomenos)		(C)–(1152)	(A)	(C)–(72)	(A)

Note the omission of the Trite element in the scale.



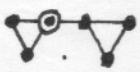


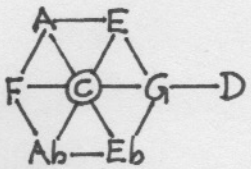
Figure 30: Some more Post Hellenistic re-interpretations of the traditional 'Music of the Planets':

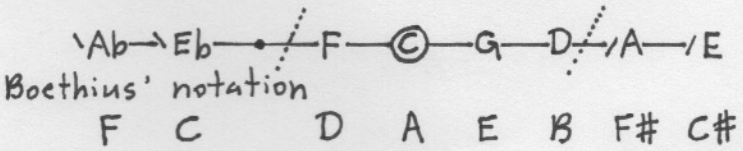
Pliny the Elder, Martianus, Censorinus, and Theon of Smyrna use two disjunct tetrachords. Rather than the usual Diatonic Tetrachords, they employ the Chromatic variety, and it has been suggested that they originally meant the Enharmonic variety. Pliny and Martianus use 'G' for the fixed stars; while Censorinus and Theon use 'F.' The two versions are so close that they are combined here:

Planet	Name of Note	Tetrachord	Pitch: 3-Lim		Pitch: 5-Lim	
✱ }	{ Nete	diezeug-menon	G - 3456	E	G - 120	E
			F - 3888	\D	F - 135	\D
♄	Paranete		/E - 4096	/C#	E♮ - 144	C#
♃	Trite		\Eb - 4374	\C	Eb - 150	C
♂	Paramese		D - 4608	B	D - 160	B
☉	Mese	Meson	C - **5184**	A	C - **180**	A
♀	Lichanos		/A - 6144	F#	A♮ - 216	\F#
☿	Parhypate		\Ab - 6561	\F	Ab - 225	F
☽	Hypate		G - 6912	E	G - 240	E
⊕	(Proslamb.)	Hypaton	F - 7776	\D	F - 270	\D
			(C) - (10,368)	(A)	(C) - (360)	(A)

Pattern on the matrix: 3-Limit

5-Limit





Achilles Tatios gives a close variant of the above scale. Note that he changes the order of the planets.

Planet	Name of Note	Tetrachord	Pitch: 3-Lim		Pitch: 5-Lim	
✱	Nete	diezeug-menon	F - 1944	\D	F - 135	\D
♄	Paranete		/E - 2048	/C#	E♮ - 144	C#
♃	Trite		\Eb - 2187	\C	Eb - 150	C
♂	Paramese		D - 2304	B	D - 160	B
☿	Mese	Meson	C - **2592**	A	C - **180**	A
♀	Lichanos		\Bb - 2916	\G	Bb - 200	G
☉	Parhypate		/A - 3072	F#	A - 216	\F#
☽	Hypate		G - 3456	E	G - 240	E
⊕	(Proslamb.)	Hypaton	F - 3888	\D	F - 270	\D
			(C) - (5184)	(A)	(C) - (360)	(A)

Like the Myth of Er monochord, the Mese is represented by Mercury (Hermes) rather than the Sun.

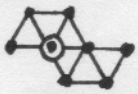


Figure 31: Cicero (1st Century A.D.) in his 'Dream of Scipio' gives his own version of the 'Planetary Music.' Ramis de Pareja (1482) interprets it as shown below. Boethius (6th Century A.D.) interprets Cicero similarly, except that he gives Venus the same value as Mercury (D), following Cicero's indications:

Planet	Name of Note	Tetrachord	Pitch: 3-Lim		Pitch: 5-Lim	
✱	Mese		C – **576**	A	C – **36**	A
♄	Lichanos	Meson	\Bb – 648	\G	Bb – 40	G
♃	Parhypate		\Ab – 729	\F	Ab – 45	F
♂	Hypate		G – **768**	E	G – **48**	E
☉	Lichanos	Hypaton	F – 864	\D	F – 54	\D
♀	Parhypate		\Eb – 972	\C	Eb – 60	C
☿	Hypate		D – 1024	B	D – 64	B
☽	Proslamb.		C – 1152	A	C – 72	A

Dio Cassius (2nd–3rd Century A.D.) gives essentially the same monochord, but with the planetary order reversed. Note that this version comes quite close to the 'original' Planetary Monochord (Double **72**:144):

Planet	Name of Note	Tetrachord	Pitch: 3-Lim		Pitch: 5-Lim	
☽	Mese		C – **576**	A	C – **36**	A
☿	Lichanos	Meson	\Bb – 648	\G	Bb – 40	G
♀	Parhypate		\Ab – 729	\F	Ab – 45	F
☉	Hypate		G – 768	E	G – 48	E
♂	Lichanos	Hypaton	F – 864	\D	F – 54	\D
♃	Parhypate		\Eb – 972	\C	Eb – 60	C
♄	Hypate		D – 1024	B	D – 64	B
✱	Proslamb.		C – 1152	A	C – 72	A

Dio Cassius gives the 3-Limit version as a sequence of fifths-fourths, showing how the planetary rulers form the proper order for the days of the week. He attributes this sequence and the attribution to the ancient Egyptians. The sequence or line of fifths, of course, is the matrix which is shown below:

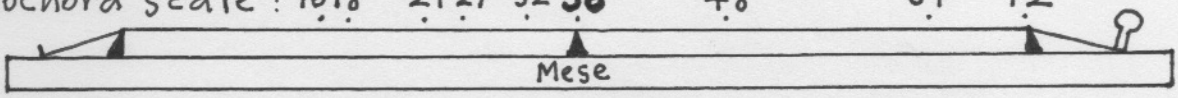
Matrix for 3-Limit Diatonic Heptad:

Planet	\Ab	\Eb	\Bb	F	©	G	D
Weekday	♀	♃	☿	♂	☽	☉	♄
	Friday	Thursday	Wednesday	Tuesday	Monday	Sunday	Saturday

Figure 32: Systems of 'Planetary Music' based on the fixed tones of the ancient Greek musical system. Both the Greater Perfect and Lesser Perfect systems are here combined. (See also **Figure 20**). The resulting harmony is the 3-Limit or Pythagorean Tetrad, which forms the matrix:

F—©—G—D

Tetrachord	Hyper.	Diez.	Synem.	Meson	Hypaton	Diazeuxis
Planetary Attribution (both directions)	⊕ ☽ / ✱ ♄	☿ ♃	♀ ♂	⊙ ⊙	♂ ♀	♃ ☿	♄ ☽	✱ ⊕
Pitch: Boethius	B A	E D	B A		E	B	A
Modern	D C	G F	D C		G	D	C
Harmonics scale	108 96	72 64	54 48		36	27	**24**
Monochord scale	16 18	24 27	32 **36**		48	64	72



Ptolemy (2nd Century), in his Harmonics (fortified by an inscription found at Canobus in Egypt) gives a scale as above, using harmonics numbers. Plutarch (1st Century AD) hints at the same construction:

Planet	Name of Note	Tetrachord	Pitch: Harmonics	Monochord	
✱			D – 36 → 108	16 (8)	B
♄	Nete Hyperbolaion	Hyperbolaion	C – 32 → 96	18 (9)	A
♃	Nete Diezeugmenon	Diezeugmenon	G – 24 → 72	24 (12)	E
♂	Nete Synnemenon	Synemmon	F – $21\frac{1}{3}$ → 64	27 ($13\frac{1}{2}$)	D
⊙	Paramese		D – 18 → 54	32 (16)	B
☿ ♀	Mese		C – 16 → 48	**36** (18)	A
☽	Hypate Meson	Meson	G – 12 → 36	48 (24)	E
Fire / Air	Hypate Hypaton	Hypaton	D – 9 → 27	64 (32)	B
Water/Earth	Proslambanomenos		C – **8** → **24**	72 (36)	A

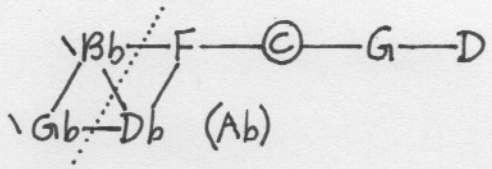
Anatolius (3rd Century) re-interprets the same number sequence (with some errors) as a monochord sequence:

Planet	Name of Note	Tetrachord	Pitch: Monochord	
⊕			D – 8 → 24	B
☽	Nete Hyperboliaon	Hyperboliaon	C – 9 → 27	A
☿	Nete Diezeugmenon	Diezeugmenon	G – 12 → 36	E
♀	Paramese		D – 16 → 48	B
⊙	Mese		C – **18** → **54**	A
♂	Lichanos Meson	Meson	A# – $21\frac{2}{3}$ → 65 ?	F×
♃	Hypate Meson	Hypaton	G – 24 → 72	E
♄	Hypate Hypaton		↓D – 34 (?) → 102	B
✱	Proslamb.		C – 36 → 108	A

Figure 33: Two unusual versions of the 'Planetary Music' from 10th Century Arabic sources

Al-Kindī gives a monochord scale which can be interpreted as both conjunct or disjunct. It does not cover a full octave. I have interpreted it here as **5**-Limit:

Planet	Name of Note	Tetrachord	Pitch (Monochord)	
☽	Nete Diezeugmenon		G – 384	E
☿		Diezeugmenon	\Gb – 405	\Eb
♀	Nete Synemmenon		F – 432	\D
☉	{Paramese	Synemmenon	D – 512	B
♂			Db – 540	\Bb
♃	Mese		C – **576**	A
♄	Lichanos Meson	Meson	\Bb – 648	\G
(*)			(Ab) – (720)	(F)
			(G) – (1152)	(A)



Ikhwān al-Ṣafā (meaning Brethren of Purity) gives a most unusual monochord scale. It appears to be a variant of the approach based on fixed tones (Figure **32**) but the numbers are altered to add a **7**-Limit element and a 13-Limit element (!). The scale still roughly follows the contour of the fixed tone approach:

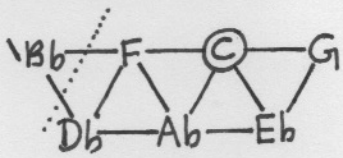
Planet	Name of Note	Tetrachord	Pitch (Monochord)	
Earth			D 8 → 112	B
Air	Nete Hyperbolaion	Hyperbolaion	C 9 → 126	A
☽	Nete Diezeugmenon	Diezeugmenon	G 12 → 168	E
☿	Nete Synemmenon	Synemmenon	F♯ 13 → 182	··D··
♀	Paramese		D 16 → 224	B
☉	Mese		C **18 → 252**	A
♂	Lichanos Meson	Meson	A♯ 21½ → 301	F#
♃	Hypate Meson		G 24 → 336	E
♄	Lichanos Hypaton	Hypaton	F♭ 27⅔ → 386	··D··
*	Hypate Hypaton		D 32 → 448	B
	Proslamb.		(C) (36) → (504)	(A)

13-Limit elements cannot be drawn on the Matrix, which is only appropriate for the 3-Limit, 5-Limit, and 7-Limit.

Figure 34: Harmonies underlying the various historical scale choices for 'Planetary music.' Some suggested planetary appellations are outlined, along with the 3-Limit and 5-Limit pitch numbers. The Boethius notation (on A) is shown along with the accurate C notation. The scale numbers are presented both as Harmonic Series componants (H) and as Monochord numbers. (M). The first octave is given in smallest whole numbers; the second octave always has at least one fractional element; the third octave is given only highlights.

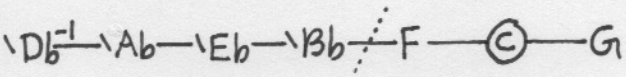
Various modes of the Diatonic Heptad:

Planetary Assignment	5-Limit Pitch: M	H		3-Limit Pitch M	H		
Nicomachus	C – 6	720	A	C – 96	3888	A	
	G – 8	540	E	G – 128	2916	E	
	F – 9	480	D	F – 144	2592	D	
	C – 12	360	A	C – 192	1944	A	
	Bb – 13½	320	G	Bb – 216	1728	G	
	Ab – 15	288	F	Ab – 243	1536	F	
⊕ ✱	G – 16	270	E	G – 256	1458	E	
☽ ♄	F – 18	240	D	F – 288	1296	D	
☿ ♃	Eb – 20	216	C	Eb – 324	1152	C	
♀ ♂	Db – 22½	192	Bb	Db^{-1} – 364½	1024	Bb	
✱ ☽ ✱ ⊙ ⊙	C – **24**	180	A	C – **384**	972	A	Mese
♀ ♄ ♂ ♀	Bb – 27	160	G	Bb – 432	864	G	
♀ ♃ ♃ ☿	Ab – 30	144	F	Ab – 486	768	F	
⊙ ♂ ♄ ☽	G – 32	135	E	G – 512	729	E	
♂ ⊙ ✱ ⊕	F – 36	120	D	F – 576	648	D	
♃ ♀	Eb – 40	108	C	Eb – 648	576	C	
♄ ☿	Db – 45	96	Bb	Db^{-1} – 729	512	Bb	
✱ ☽	C – 48	**90**	A	C – 768	**486**	A	



Double **24**:48

Dual **90**:180

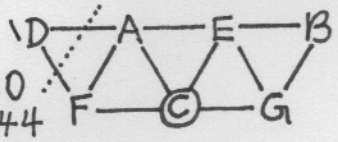


Double **384**:768

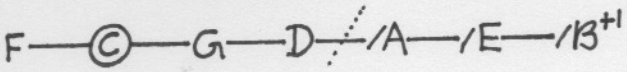
Dual **486**:972

The demonstration of modes of the Diatonic continues over pages. After the 7 Diatonic Heptad modes comes the 8 modes of the Diatonic Ogdoad.

Planetary Assignment	5-Limit Pitch	M	H	3-Limit Pitch	M	H	
Babylonian	C –	7½	576	C –	121½	3072	A
	G –	10	432	G –	162	2304	E
	E –	12	360	/E –	192	1952	C#
	C –	15	288	C –	243	1536	A
	B –	16	270	/B+1 –	256	1458	G#
	A –	18	240	/A –	288	1296	F#
⊕ ✳	G –	20	216	G –	324	1152	E
☽ ♄	F –	22½	192	F –	364½	1024	D
☿ ♃	E –	24	180	/E –	384	976	C#
♀ ♂	\D –	27	160	D –	432	864	B
* ☽ ✳ ☉ ☉	C –	**30**	144	C –	**486**	768	A
☿ ♄ ♂ ♀	B –	32	135	/B+1 –	512	729	G#
♀ ♃ ♃ ☿	A –	36	120	/A –	576	648	F#
☉ ♂ ♄ ☽	G –	40	108	G –	648	576	E
♂ ☉ ✳ ⊕	F –	45	96	F –	729	512	D
♃ ♀	E –	48	90	/E –	768	486	C#
♄ ☿	\D –	54	80	D –	864	432	B
* ☽	C –	60	**72**	C –	972	**384**	A

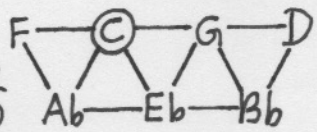


Double **30**:60
Dual **72**:144

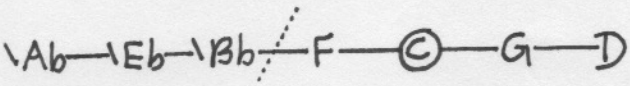


Double **486**:972 Dual **384**:768

Dio Cassius, Plutarch, Ptolemy, Plato, Nicomachus, Cicero	5-Limit Pitch:	M	H	3-Limit Pitch	M	H	
	C –	9	960	C –	144	5184	A
	G –	12	720	G –	192	3888	E
	Eb –	15	576	\Eb –	243	3072	C
Boethius	C –	18	480	C –	288	2592	A
Euclid, Archytas	Bb –	20	432	\Bb –	324	2304	G
	Ab –	22½	384	\Ab –	364½	2048	F
⊕ ✳	G –	24	360	G –	384	1944	E
☽ ♄	F –	27	320	F –	432	1728	D
☿ ♃	Eb –	30	288	\Eb –	486	1536	C
♀ ♂	D –	32	270	D –	512	1458	B
* ☽ ✳ ☉ ☉	C –	**36**	240	C –	**576**	1296	A
☿ ♄ ♂ ♀	Bb –	40	216	\Bb –	648	1152	G
♀ ♃ ♃ ☿	Ab –	45	192	\Ab –	729	1024	F
☉ ♂ ♄ ☽	G –	48	180	G –	768	972	E
♂ ☉ ✳ ⊕	F –	54	160	F –	864	864	D
♃ ♀	Eb –	60	144	\Eb –	972	768	C
♄ ☿	D –	64	135	D –	1024	729	B
* ☽	C –	72	**120**	C –	1152	**648**	A

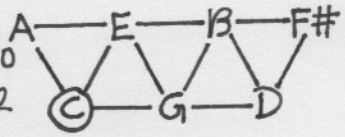


Double **36**:72
Dual **120**:240

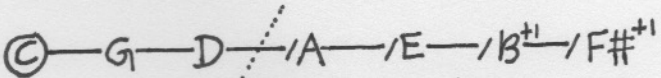


Double **576**:1152 Dual **648**:1296

Planetary Assignment	5-Limit Pitch	M	H	3-Limit Pitch	M	H	
	C –	11¼	768	C –	182¼	4096	A
	G –	15	576	G –	243	3072	E
	D –	20	432	D –	324	2304	B
	C –	22½	384	C –	364½	2048	A
	B –	24	360	/B^{+1} –	384	1944	G#
	A –	27	320	/A –	432	1728	F#
⊕ ✳	G –	30	288	G –	486	1536	E
☽ ♄	F# –	32	270	/F#$^{+1}$ –	512	1458	D#
☿ ♃	E –	36	240	/E –	576	1296	C#
♀ ♂	D –	40	216	D –	648	1152	B
✳ ☽ ✳ ⊙ ⊙	C –	**45**	192	C –	**729**	1024	A
☿ ♄ ♂ ♀	B –	48	180	/B^{+1} –	768	972	G#
♀ ♃ ♃ ☿	A –	54	160	/A –	864	864	F#
⊙ ♂ ♄ ☽	G –	60	144	G –	972	768	E
♂ ⊙ ✳ ⊕	F# –	64	135	/F#$^{+1}$ –	1024	729	D#
♃ ♀	E –	72	120	/E –	1152	648	C#
♄ ☿	D –	80	108	D –	1296	576	B
✳ ☽	C –	90	**96**	C –	1458	512	A

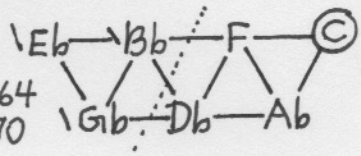


Double **45**:90
Dual **96**:192

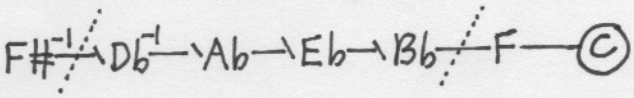


Double **729**:1458 Dual **512**:1024

Planetary Assignment	5-Limit Pitch	M	H	3-Limit Pitch	M	H	
	C –	8	1080	C	128	5832	A
	F –	12	720	F	192	3888	D
	Db –	15	576	\Db^{-1}	243	3072	Bb
	C –	16	540	C	256	2916	A
	\Bb –	18	480	\Bb	288	2592	G
	Ab –	20	432	\Ab	324	2304	F
⊕ ✳	\Gb –	22½	384	F#$^{-1}$	364½	2048	Eb
☽ ♄	F –	24	360	F	384	1944	D
☿ ♃	\Eb –	27	320	\Eb	432	1728	C
♀ ♂	Db –	30	288	\Db^{-1}	486	1536	Bb
✳ ☽ ✳ ⊙ ⊙	C –	**32**	270	C	**512**	1458	A
☿ ♄ ♂ ♀	\Bb –	36	240	\Bb	576	1296	G
♀ ♃ ♃ ☿	Ab –	40	216	\Ab	648	1152	F
⊙ ♂ ♄ ☽	\Gb –	45	192	F#$^{-1}$	729	1024	Eb
♂ ⊙ ✳ ⊕	F –	48	180	F	768	972	D
♃ ♀	\Eb –	54	160	\Eb	864	864	C
♄ ☿	Db –	60	144	\Db^{-1}	972	768	Bb
✳ ☽	C –	64	**135**	C	1024	**729**	A

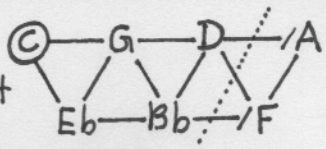


Double **32**:64
Dual **135**:270



Double **512**:1024 Dual **729**:1458

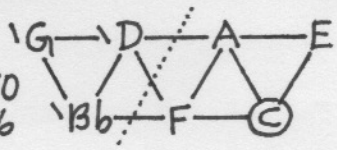
Planetary Assignment	5-Limit Pitch	M	H	3-Limit Pitch	M	H	
Anatolius	C –	6¾	640	C –	108	3456	A
	G –	9	480	G –	144	2592	E
	D –	12	360	D –	192	1944	B
	C –	13½	320	C –	216	1728	A
	Bb –	15	288	\Bb –	243	1536	G
	/A –	16	270	/A –	256	1458	F#
⊕ ⚹	G –	18	240	G –	288	1296	E
☽ ♄	/F –	20	216	F –	324	1152	D
☿ ♃	Eb –	22½	192	\Eb –	364½	1024	C
♀ ♂	D –	24	180	D –	384	972	B
* ☽ ⚹ ☉ ☉	C –	**27**	160	C –	**432**	864	A
☿ ♄ ♂ ♀	Bb –	30	144	\Bb –	486	768	G
♀ ♃ ♃ ☿	/A –	32	135	/A –	512	729	F#
☉ ♂ ♄ ☽	G –	36	120	G –	576	648	E
♂ ☉ ⚹ ⊕	/F –	40	108	F –	648	576	D
♃ ♀	Eb –	45	96	\Eb –	729	512	C
♄ ☿	D –	48	90	D –	768	486	B
⚹ ☽	C –	54	**80**	C –	864	**432**	A



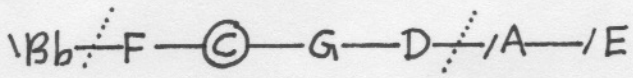
Double **27**:54
Dual **80**:160

Double **432**:864 Self-dual.

Planetary Assignment	5-Limit Pitch	M	H	3-Limit Pitch	M	H	
	C –	10	864	C –	162	4608	A
	A –	12	720	/A –	192	3888	F#
	E –	16	540	/E –	256	2916	C#
	C –	20	432	C –	324	2304	A
	\Bb –	22½	384	\Bb –	354½	2048	G
	A –	24	360	/A –	384	1944	F#
⊕ ⚹	\G –	27	320	G –	432	1728	E
☽ ♄	F –	30	288	F –	486	1536	D
☿ ♃	E –	32	270	/E –	512	1458	C#
♀ ♂	\D –	36	240	D –	576	1296	B
* ☽ ⚹ ☉ ☉	C –	**40**	216	C –	**648**	1152	A
☿ ♄ ♂ ♀	\Bb –	45	192	\Bb –	729	1024	G
♀ ♃ ♃ ☿	A –	48	180	/A –	768	972	F#
☉ ♂ ♄ ☽	\G –	54	160	G –	864	864	E
♂ ☉ ⚹ ⊕	F –	60	144	F –	972	768	D
♃ ♀	E –	64	135	/E –	1024	729	C#
♄ ☿	\D –	72	120	D –	1152	648	B
⚹ ☽	C –	80	**108**	C –	1296	**576**	A



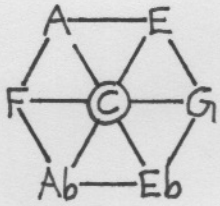
Double **40**:80
Dual **108**:216

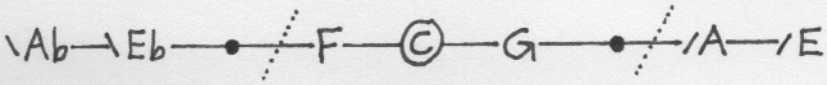


Double **648**:1296 Dual **576**:1152

The Harmonic Heptad as source for 'Planetary Music':

Planetary Assignment	5-Limit Pitch	M	H	3-Limit Pitch	M	H	
Pliny	C –	15	480	C –	1296	41472	A
Martianus	G –	20	360	G –	1728	31104	E
Censorinus	F –	22½	320	F –	1944	27648	D
Theon of Smyrna	C –	30	240	C –	2592	20736	A
Achilles Tatios	A –	36	200	/A –	3072	17496	F#
	Ab –	37½	192	\Ab –	3280½	16384	F
⊕ ⚹	G –	40	180	G –	3456	15552	E
☽ ♄	F –	45	160	F –	3888	13824	D
☿ ♃	E –	48	150	/E –	4096	13122	C#
♀ ♂	Eb –	50	144	\Eb –	4374	12288	C
* ☽ ⚹ ☉ ☉	C –	**60**	120	C –	**5184**	10,368	A
☿ ♄ ♂ ♀	A –	72	100	/A –	6144	8748	F#
♀ ♃ ♃ ☿	Ab –	75	96	\Ab –	6561	8192	F
☉ ♂ ♄ ☽	G –	80	90	G –	6912	7776	E
♂ ☉ ⚹ ⊕	F –	90	80	F –	7776	6912	D
♃ ♀	E –	96	75	/E –	8192	6561	C#
♄ ☿	Eb –	100	72	\Eb –	8748	6144	C
⚹ ☽	C –	120	**60**	C –	10,368	**5184**	A





Double **60**:120
Self-dual

Double **5184**: 10,368
Self-dual

Figure 35: The number series underlying the cosmological heptads **24**:48, **30**:60, and **36**:72 and the tetrachord frames of Ptolemy, Plutarch, Anatolius:

Mese	6	8	9	10	12	15	16	18	20	24	27	30	32	36	40	45	48	54	60	64	72
	Monochord Sequence																				
18		D	C		G	13½ F	D	**C**		G	F			**C**	Anatolius						
24	C	G	F	Eb	C	Ab	G	F	Eb	**C**	\Bb	Ab	G	F	Eb	Db	C				
30	E	B	A	G	E	C	B	A	G	E	\D	**C**	B	A	G	F	E	\D	C		
36	G	D	C	Bb	G	Eb	D	C	Bb	G	F	Eb	D	**C**	Bb	Ab	G	F	Eb	D	C
	Harmonics Sequence																				
8		**C**	D		G		C	D	F	21⅓ G			C	D	Ptolemy, Plutarch						
24										**C**	D	E	F	G	A	B	C	D	E	F	G
30												**C**	Db	Eb	F	G	Ab	Bb	C	Db	Eb
36														**C**	\D	E	F	G	A	\Bb	C
	6	8	9	10	12	15	16	18	20	24	27	30	32	36	40	45	48	54	60	64	72

Figure 36: The seven classical Greek Harmoniai or Heptatonic diatonic modes

Monochord Division (L = Limit)	Pitch range as an "Octave Species" A-Mese	C-Mese	Modern name after 1547 Glareanus	Classical Greek name
5-L { **72**:144 **24**:48 3-L **384**:768	E	G	Phrygian	Dorian *
5-L **90**:180 3-L **432**:864	D	F	Dorian	Phrygian *
5-L { **90**:180 **30**:60 3-L **486**:972	C	Eb	Ionian	Lydian
5-L **96**:192 3-L **512**:1024	B	D	Locrian	Mixolydian, (Hyperdorian)
5-L **36**:72 3-L **576**:1152	A	C	Aeolian	Hypodorian (Aeolian)
5-L **120**:240 3-L **648**:1296	G	Bb	Mixolydian	Hypophrygian (Ionian)
5-L **45**:90 3-L **729**:1458	F	Ab	Lydian	Hypolydian

Pitch range on the monochord of the Gk. Dorian and Phrygian interpreted as an "octave species"

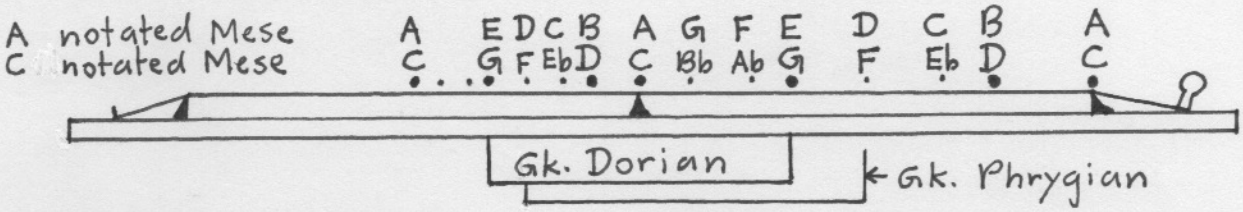
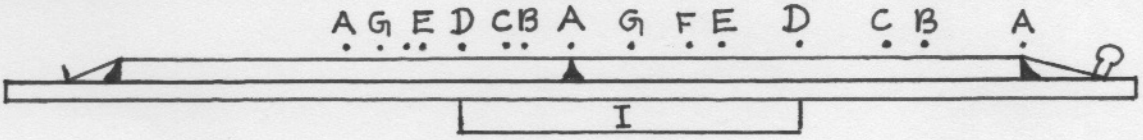


Figure 37: The early Christian Church Modes, four "Authentic" which are odd-numbered and are attributed to St. Ambrose (340-97 A.D.), and four "Plagal" which are even-numbered and attributed to pope Gregory (540-604 A.D.). All are 3-Limit and based on monochord **576**:1152.

Four Authentic Modes				Four Plagal (Derived) Modes			
Name	Octave Species	Dom.	Modern Name	Name	Octave Species	Dom.	Modern Name
I	D	(A)	Dorian	II	A	(F)	Aeolian
III	E	(C)	Phrygian	IV	B	(A)	Locrian
V	F	(C)	Lydian	VI	C	(A)	Ionian
VII	G	(D)	Mixolydian	VIII	D	(C)	Dorian