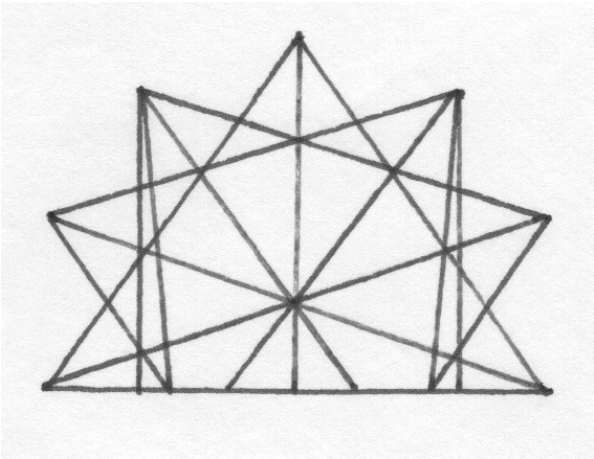
The Meantone Series of Cyclical Temperaments

an introduction and comparative appraisal



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AN INTRODUCTION AND COMPARATIVE APPRAISAL

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A cyclical temperament is a tuning system which may be expressed as a closed circle of Fifths. Such tunings have the advantage of unlimited modulation of harmonic pattern, that is, the patterns are invariant under transposition. But we must pay a price for this mobility. The drawback lies in the mandatory use of irrational ratios instead of small-integer "Just" ratios. Such tuning systems use controlled "mis-tunings" of the normative simple ratios. Hence they are called Temperaments.

These temperaments are also necessarily Equal Temperaments since they must make their patterns from the aggregates of some basic step or fundamental ratio. The basic step is some multiple root of 2 and hence an irrational number. We judge the system by observing how close the tempering is to the Just ratio.

It is well known that certain Equal-Tempered systems are useful because they provide close enough approximations to the Just intervals to "trick" our ears into believing that we are in tune. Hence we put up with the slight innaccuracy in order to take advantage of the structural mobility of the system. The best of such systems are the division of the octave into 12, 19, 31, 43, and 53.

The 53 system stands alone in preserving practically all of the properties of Just Intonation. Thus it is named Quasi-Just Intonation. Practically speaking, it is a scale of Commas. It can be represented as a tuning lattice showing the interaction of the 3rd Harmonic and the 5th Harmonic in tuning, identical to the lattice of Just Intonation.

The other systems are best represented as alterations of this lattice. In this way the "personality" of the system is clarified and a harmonic syntax natural to the system is suggested. The graphic approach which I employ elucidates the "quality" of the system and simplifies our comprehension of its structure.

The purpose of this essay is to introduce these prominant systems and present a comparative appraisal.

JUST INTONATION

We must begin with Just Intonation since other tuning systems are considered modifications of it. The rules of the tuning game are these: in the layout of our scale we use pure musical Fifths (ratio 3:2) and pure musical Thirds (ratios 5:4 and

6:5). We tune strings to each other by eliminating "beats" between the harmonics of the strings concerned. In effect we are following Plato's marriage allegory (from the Republic) in "mating 4:3 with 5." The result is a tuning lattice or field derived directly from the 3 times 5 multiplication table.

The nucleus of this lattice is shown in Figure 1. The three axes of Just Intonation result in six tuning directions. The horizontal axis (the 3:2 axis) demonstrates tuning in Just musical Fifths and Fourths. We will arbitrarily use the pitch "C" as our reference pitch. Then to tune "G" from "C" we may tune up a 3:2 ratio or tune down a 3:4 ratio. Downward tunings are bracketed. Any horizontal movement to the right indicates such a tuning operation which is in the "Dominant" direction. We may tune in the opposite direction to produce the inversion. Thus we may tune "F" by tuning up in pitch a musical Fourth 4:3, or down a Fifth (2:3). Now we are tuning in the "Sub-dominant" direction.

The second axis of Just Intonation, the 5:4 axis, allows us to tune Just Major Thirds and Just Minor Sixths. The same tuning proceedures as above apply. This axis takes us upward to the right and downward to the left. Note that the upward pointing triangle (C-E-G) is the Just Major Triad which employs the first six harmonics. Every upward pointing triangle is a Just Major Triad.

The third axis of Just Intonation, the 5:3 axis, points upward to the left, and downward to the right. It demonstrates tuning in Just Major Sixths and Just Minor Thirds. Note that the downward pointing triangle (A-C-E) is the Just Minor Triad. Every such shape indicates a Just Minor Triad.

These six tones (E-G-Eb-Ab-F-A) represent the closest harmonic relation to the generator tone C. In Helmholtz' book, ON THE SENSATIONS OF TONE the translator Ellis names this pattern a UNIT OF HARMONY. It is the fundamental unit of the Just Intonation lattice or field.

In Figure 1 on the right the pitch names are converted into Comma numbers. Since Just Intonation may be seen conveniently as a scale of commas, the pitches may be numbered from 0 (C) up to 52. Pitch 53 equals pitch 0. Here is a handy notation which also is useful in indicating the size of an interval in commas. Thus the interval of the musical Fifth is 31 commas in size, and the just Major Third is 17 commas in size. Many of the diagrams of this essay use the Comma Notation instead of the pitch notation due to its simplicity. Pitch name and Function are also used.

These tuning operations may be extended in all six directions from our reference pitch C, resulting in the expanded field of Just Intonation shown in ratios as Figure 13, and in pitch names as Figure 18. Note that this expansion may also be viewed in functional terms so that we have the field of Functions pictured in Figure 19. Also note that 53-E.T. uses the same pitch names and functional names as Just Intonation.

as its Just Major Third. We are confronted with alternative forms of wholestep and Minor Seventh.

We will maintain our "strict" horizontal expansion through this boundary and tune the Pythagorean Heptad--the line of pitches 13-44-22-0-31-9-40. We now have the Pythagorean version of the Diatonic Scale, in it's symmetrical mode, the Dorian. Beyond the Meantone Corridor boundary we have expanded into the comma-raised region (ie. pitch 40) and the comma-lowered region (ie. pitches 44 and 13). We have here another ancient Greek and Chinese scale.

One more expansion brings us to the Pythagorean Ennead consisting of pitches 35-13-44-22-0-31-9-40-18. We are again at an important boundary region of Just Intonation. If we add a 10th tone in the sub-dominant direction we note that the pitch, 4, is underlined. This symbol means that it is a schisma flat of the pitch 4 which is inside the boundary (above pitch 40 and 18). In a similar fashion, the expansion in the Dominant direction from 18 to 49 gives us a pitch which is raised by a schisma.

This important interval is only about 2 Cents (more accurately 1.954) and is thus on the boundary of our pitch discrimination threshold. According to Seashore's standard reference work, THE PSYCHOLOGY OF MUSIC, the average threshold is about 10 Cents (that is, about one half of a comma). In my tuning experience, I find the threshold closer to somewhere between one-third of a comma (about 7 Cents) and one-quarter of a comma (about 5 Cents). Yet under ideal conditions, (that is, a long sustain and a favourable timbre) I have heard the schisma interval which is about one-twelfth of a comma (2 Cents).Yet this interval is so small as to be practically negligible under practical musical situations. If the octave were divided into schismas we would have about 614. One of the very best cyclical temperaments is the 612-E.T. division, but it is, of course, totally impractical. It is interesting that the logarithmic expression of the ratios which Ellis termed Cents is a division into 1200, which relates quite well to the schisma interval.

The schisma interval has great theoretical and practical importance in the structure of Just Intonation. For the first time in practical tuning, we have encountered the subliminal. It is the nature of the 53-E.T. system to temper away the schisma so that only the comma remains. But for now we will maintain our schisma in tact and see what happens to the Field.

Continuing the examination of our Ennead (Figure 3) we see that the end pitches 18 and 35 relate by a Just Major Third flattened by a schisma. For a Just Major Third above 18 is pitch 35 across the right boundary. And on the other side, a Just Major Third below 35 is pitch 18. Hence we must come to the conclusion that any long expansion along the Pythagorean axis results in intervals which are not "Pythagorean" but "Just". Now these intervals are not strictly Just, since they are schisma-altered versions of Just, but the sonic effect is practically identical.

Tuning innacuracies in producing a long line of eight Musical Fifths may already result in over 2 Cents innacuracy. Thus we may conclude that there are two ways of tuning pitch 17 (E) from 0 (C). We can tune a Just 5:4 directly, or we can tune a line of eight fifths from 0-22-44-13-35-4-26-48-to 17. Strictly speaking, this second method gives us 17 flattened by a schisma.

Let us look at a chromatic expansion of the Pythagorean horizontal axis. On Figure 5 we see a tuning of Fourths-Fifths in the Dominant direction from 0 through twelve Fifths to produce the 13th tone, pitch $\overline{1}$. We have produced the Syntonic Comma raised by a schisma--an interval called the Pythagorean Comma or the Ditonic Comma. If we restrict the expansion to 12 tones (and hence an assymmetrical expansion) we have a pattern which contains an extensive eight note segment of "Just Intonation" called the Ptolemaic Ogdoad. This prominant pattern of Just Intonation is also pictured at the bottom of Figure 7 and the top of Figure 12. In Figure 5 we see it centred on pitch 14 in its schisma-altered form. This pattern contains the two principal forms of the Just Diatonic Scale, called the Ptolemaic Heptads, in Figure 7.

Hence we must conclude that a chromatic expansion of the Pythagorean axis results in a chromatic scale which embodies an essential polarity. One eight note segment of it contains the Ptolemaic Ogdoad, and another eight note segment of it contains the maximim expansion of Pythagorean intervals before encountering a "Just" interval, the Pythagorean Ogdoad. The structural differences between these two eight tone patterns are shown on Figure 12.

A short introduction to the CIRCULAR GRAPH. It is a visual logarithmic transformation of the ratios, related to the Cents system. Any interval in Cents may be converted to degrees of arc by multiplying it by 0.3. The octave interval is defined as 360 degrees of arc. Counter-clockwise movement shows the raising of pitch, and clockwise movement the lowering of pitch. The relationship (interval) between two pitches appears visually as the "chord" or line connecting the two pitches. Thus we have a visual analog of any harmonic pattern which is useful for examining the "metrical" properties of the intervals. The circular graph also indicates whether a pattern is symmetrical or not, and where its axis of symmetry lies. It also gives other information which we won't consider right now.

Figure 12 contrasts the interval structure of the Ptolemaic Ogdoad and the Pythagorean Ogdoad. Remember that both patterns are inherent in any chromatic Pythagorean tuning. Thus, returning to Figure 5, the line of Fifths shown at the top may be re-written in the two forms shown below it (which display the Ptolemaic Ogdoad). Any long line of Fifths may be re-defined as two lines of Fifths relating by Just intervals.

Since the schisma is practically subliminal, this chromatic scale could also be tuned in various schisma-altered variations which are practically identical in sound. Their ratio

numbers may differ radically but in sound they are the same. For instance, we can take the top chromatic scale and lower the fifth between 18 and 49 by a schisma, so that we have fifth 18-49. Then we carry on with our line of pure Fifths producing the scale pictured below as one of the many schisma-altered variations. As soon as we tune a schisma-flattened fifth in any line of pure Fifths, we create a "second line" of Fifths, as shown.

In the middle of Figure 5 we see the tuning of a line of Fifths-Fourths from 39 (A) through to 9 (D) with a schisma-flat fifth between 35 and 13. This is a pleasing form of the scale since it places the Ptolemaic Ogdoad in a prominant position and uses the smallest ratio numbers. We will call this pattern a "Just Pythagorean" scale. Remember that any Pythagorean chromatic scale has 12 modes, two of which place either the Pythagorean Ogdoad or the Ptolemaic Ogdoad in central positions. The scale shown at the top of Figure 5 has a Ptolemaic Ogdoad centred in the Key of Eb (14).

The bottom of Figure 5 shows various patterns all of which have the same essential sound quality (if we disregard schismas). The "tonic" (0) may be placed in any position on these patterns. Thus there are many "types" of Pythagorean chromatic scales. The alteration of these various patterns lead to the many forms of 18th Century tunings called "Well-Temperaments."

But let us carry on and maintain our strict "Pythagoreanism." Figure 6 shows the resulting harmonic pattern comprised of a line of 16 Fifths--the 17 tones 52-30-8-...-9-40-18. This mediaeval Arabic tuning shows a maximum extension of the Ptolemaic Sequence, a Just tuning series outlined on Figure 7. We now have a scale with both the Just Major Third (17) and the Pythagorean Major Third (18). The chromatic scale has alternative forms of degree steps.

The 25 tone symmetrical expansion shown below shows a very important "boundary". For now all the tones of the chromatic scale have both a Just form and a Pythagorean form. 22 of these pitches comprise the Hindu Sruti tuning system. We see that the Vedic system, as well as the Chinese system, has its roots in the Pythagoreanism which probably originated in the nuclear Near-East (Sumer and Babylon).

We can theoretically continue our symmetrical expansion of the horizontal axis toward infinity, but an important boundary is reached after the 53rd Fifth. We reach the ENHARMONIC GATEWAY between pitches 11 and 42. These two pitches also appear when we tune an expanded string of Minor Thirds-Major Sixths along the 5:3 axis--ie. 11-25-39-0-14-28-42. The dotted intervals above are raised about 8.1 Cents, and the intervals shown at the lower end are lowered the same amount. A northern and southern boundary is created in the Field. We will examine this important region more latter when we look at the structure of 53-E.T. and 19-E.T. I wish now to clarify another region of Just Intonation, showing the

relation of Just Intonation and intervals which use the 7th Harmonic--the important Septimal ratios 7/4, 7/5, 7/6, and so on.

Since the Just lattice is made up of powers of the 3rd and 5th Harmonics, the Septimal ratios are not represented. However, Figure 9 shows that certain 5-Limit ratios approximate the 7-Limit ratios. For example, the Septimal ratio 7/6 is approximated by ratio 75/64, which is about 7.7 Cents sharp of the Septimal consonance. The triad pattern C-D#-G is therefore the Septimal Minor (or Sub-minor) Triad in sound, even though it is slightly "off-tune". Seven Cents is about a third of a comma and is "almost" subliminal. In the upper diagram, this pattern contains within it the HARMONIC SERIES PENTAD , since C is the fundamental, G is the 3rd harmonic, E is the 5th harmonic, A# is the 7th harmonic, and D is the 9th harmonic. The Harmonic Series is not symmetrical, so it has a "mirror" in the Sub-harmonic series shown below. Thus the Just Intonation lattice also graphs the important Septimal intervals in a "hidden" way. In the ancient number philosophy, the number 7 was a number of mystery. We will find this number very important when examining the Meantone Cyclical Temperaments.

We have looked at expansion along the 3:2 axis, and the 5:3 axis, but we have not touched the 5:4 axis. For now, observe the tuning progression 34-17-0-36. Pitch 34 is G#, and pitch 36 is Ab. They relate by an interval of about two commas in size, called the DIESIS, or enharmonic. This interval is the difference between a sharp and a flat. This interval gains prominance in 31-E.T.

I will conclude this short introduction to the regions of the Just Intonation Field with a brief consideration of the HARMONIC ANTIPODES. We see this peculiar region prominantly displayed, along with the regional boundary of 53-E.T., on Figure 16. The four pitches 24, 29, 7, and 46 seem to inhabit all four "corners" of the Field. No matter in which direction one tunes, except for the Enharmonic Gateway, one ends up at the Harmonic Antipodes. If our tuning reference 0 is the North Pole, then walking in any direction eventually takes up to the South Pole, the Antipodes. The four corners are only subliminally different from each other.

Pitch 24 (Gbb) is a 5-Limit approximation of the 11th Harmonic (ratio 11:8). Its inversion pitch 29 (Fx) represents the 11th Subharmonic (ratio 16:11). Pitches 7 and 46 show neutral Seconds and Seventh intervals. The closely juxtaposed pitches 15 and 38 indicate neutral Thirds and Sixths. Certain Cyclical Temperaments preserve the Antipodes, and others, notably 12-E.T. and 19-E.T., abolish it altogether. The Antipodes are most closely associated with 53-E.T. and 31-E.T.

53-E.T. (QUASI-JUST INTONATION)

It is natural to consider this Cyclical Temperament next since it is so closely associated with Just Intonation. In fact it

sets up a very good imitation of the Just Field, the deviation, in Cents, shown as Figure 17. This tuning system eliminates the schisma, leaving a scale of commas. Remember the schisma which resided between the Syntonic Comma (21.5 Cents) and the Ditonic Comma (23.4 Cents)? Now 53-E.T. uses a "Mean Comma" of about 22.6 Cents. The distinction between the two Commas is replaced by an "aloquat" comma which is the 53rd root of 2. The octave is divided into 53 equal-sized steps.

The pitch names and Functions of Just Intonation are essentially preserved, without the schisma signs, and, I might add, without the dot signs for pitches beyond the Enharmonic Gateway. We now have a closed cycle of Fifths, pictured in Figure 11. Note on this diagram that the pitches associated with the Enharmonic Gateway, 11 and 42, lie directly opposite the generator pitch 0. Beside figure 10 and Figure 11 is the pitch notation of 53-E.T. written out beside the Comma Number notation.

You will notice that some pitches require more than one name. The reason is due to the presence of a boundary region for the temperament. For example, pitch 4 has two names in pure Just intonation. The pitch 4 on the left is \Db. Pitch 4 inside the boundary on the right is /C#. Now in 53-E.T. pitch 4 must stand for both pitches and functions. Hence it has two names. All such boundary pitches have two names. In fact, some pitches around the Harmonic Antipodes have as many as four names, for obvious reasons. All tempered tuning systems have boundaries which create functional ambiguity. This ambiguity lies in different places within different systems, but is a characteristic feature of all cyclical temperaments.

53-E.T. is a happy compromise which approximates the Pythagorean ratios extremely accurately, as well as the Just ratios to generally better than 2 Cents. Note that the approximation of the Septimal ratios is also improved over Just Intonation, the 7:4 ratio being sharp by 7.7-2.9=4.8 Cents. Other Septimal ratios are similarly improved. Its accuracy as a closed cycle of Fifths is excellant: Tuning 53 Just Fifths up in pitch and 31 octaves down in pitch creates a "near miss" of only 3.615 Cents. This error, divided among all the Fifths, means that each Fifth is flattened by about 0.068 Cents, a truly negligible amount. Note that the Major Third is flattened by 1.4 Cents, also a negligible amount.

Although an exemplary approximation of Just Intonation results, we must remember that all the ratios are irrational, and that no ratio is perfectly Just except the octave. But we have simplified the Field from over 600 pitches to the octave to only 53! All of those schisma-altered Pythagorean scales which were practically identical to each other described above are now the one and the same pattern--the Pythagorean Chromatic scale. Apart from the schisma all other Just Intonation characteristics are preserved. We still have Harmonic Antipodes and the Enharmonic Gateway.

Another characteristic feature of the Just lattice is preserved. For the Field is still planimetric. Every type of chord or scale has its own "shape" which describes its harmonic quality or essence. Any such pattern may be transposed by simply shifting it around the Field. In this way we can observe the 53 "modes" of every pattern. I have used this structural feature to compile the MODAL TABLES for the tuning system.

This tuning system allows accurate representation of all of the Raga Tunings from the Hindu music culture, as well as all of the historical Chinese scales. It was "discovered" in the West by Mercator in the 16th Century, but already described earlier by the Chinese Theorist King Fang in 306 A.D. The Chinese have always been strictly Pythagorean (3-Limit) in their tunings, and they also worked out the specifics of 12-E.T. independent of the West. However, I claim that the 53 system was known by the Pythagorean School. Philolaus, a direct disciple of Pythagoras, stated (as mentioned by Boethius) that the wholetone (ratio 9:8) is divisible into nine commas. Such a division characterizes the 53 system.

It is clear that 53 accomplishes a good mimick of vocal "a cappella" intonation and string intonation. For studies have shown that string quartets, orchestras, etc. tend to the Pythagorean intervals. When we mix Pythagorean intervals and Just intervals we have 53--and they always appear mixed in practice. Thus 53 may be considered as the "subconcious" or "intuitive" intonation of choice for much of the music produced, both past and present. The other historically important temperaments, 12-E.T. and 31-E.T. are deviations from the 53 Field, although we shall see that the 31 system is still a good approximation. The 12-E.T. system did not become "universal" until the middle of the 19th Century, and there is still discontent with its badly tempered Thirds and Sixths. The dominance of the 12-E.T. system is associated with the 19th Century dominance of the piano. The modern development of electronic music and computer-music technology will open up 53 and other systems for the keyboard. Although the system cannot be applied to fretted instruments, still it is "in the ears" of most musicians on the sub-concious level. Its basic interval, the comma, can be heard by most people, even non-musicians.

The complexity of the 53 system results in a rich assortment of chord types and scale types--too many to rigorously pursue in this essay. As an example of a triadic dissonance, I refer you to the "Just Major Scale" which is the Ptolemaic Heptad (1) shown near the bottom of Figure 7. It has an inventory of three Major Triads (on C, F, and G) and two Minor triads (on A and E) in their Just consonant form. But look at pitches 9 (D), 22 (F), and 39 (A). They form a dissonant Minor Triad, whose interval makeup is 0-13-30. It has the "dark" Pythagorean Minor Third and the dissonant comma-lowered Fifth. This dissonance "demands" the resolution to the Tonic (0-17-31). It creates a tension which properly requires resolution. The diminished triad present in pitches 9-22-48 also acts in a similar manner. The proper

resolution of the various dissonances of the system leads naturally to a syntax proper to Just Intonation.

Incidently, observing the Ptolemaic Heptad ② which is the reciprocal (inversion)of the Heptad ①, we can see a corresponding dissonant Major Triad between pitches 31 (G), 48 (B), and 8 (\D). This Major Triad also has the dissonant comma-flattened Fifth. The playing of this chord naturally leads to a resolution into the Minor Triad on A (39-0-17). Many such "directional" dissonances exist within Just Intonation and 53-E.T.

A detailed study of 53-E.T. syntax and function (mode) is not possible in this paper. It is properly the subject of another work. My task at hand is to survey the Meantone series of Cyclical Temperaments which modify the Just lattice by fractional parts of a comma. Armed with the above descriptive tools, we can now intelligently describe the various temperaments.

12-E.T. (ONE-TWELFTH DITONIC COMMA MEANTONE)

I begin the examination of the temperaments proper with the familiar 12-E.T., not due to any innate sonic superiority on its part, but rather due to its simplicity in structure. It is the smallest numbered cyclical temperament to be considered "decent" for practical use, although 7-E.T. and 5-E.T. have approximate variants in south-east Asian music.

12-E.T. is also the most "horizontal" of temperaments, being a tempering of the Pythagorean (3-Limit) line of Fifths-Fourths. Observe on Figure 20 that a symmetrical expansion of the horizontal axis to 13 tones (ie. 12 tunings of a 3:2) produces a chromatic scale with two forms of the tritone, pitch 26 (F#) and pitch 27 (/F#) at the two ends. As shown above, these two pitches relate by a Ditonic Comma. Since 12 Fifths lead to the comma, why not "chop" one-twelfth of a Comma off of each Fifth. Then the two ends will meet to form a circle of tempered Fifths. This is done starting from our tuning reference 0 (C). Pitch G is flattened 1/12th of a comma. Therefore its inversion, 22 (F) is sharpened by 1/12th of a comma. Continuing this operation, the pitch 9 (D) will be 2/12ths=1/6th of a comma flat, and its inversion correspondingly sharp. The superscripts on Figure 20 indicate the fractional parts of the comma by which the Just ratios are altered. Looking at the ends, Pitch 27 is flattened one-half of a comma, while pitch 26 is raised one-half comma. Therefore they become the same pitch, leading to a tritone which divides the octave exactly in half, the Geometric Mean of the Octave--the square root of 2. The circle is closed.

Here we have a truly elegant tuning system which reduces the complex Just Intonation Field into only 12 steps. Figure 21 shows a metrical comparison between 53-E.T. and 12-E.T. with the approximation of the Pythagorean Ogdoad filled in. This elegance, however, has its price, as shown on Figure 23.

The intervals of the Fifth and Fourth are excellant, being mis-tuned by only 1/12th of a comma. This interval is only about 2 Cents--you guessed it, a schisma! The Wholetone and Minor Seventh (in its Pythagorean form) are mis-tuned by only 1/6th of a comma, still practically subliminal. But now alas, observe the Major Sixth. The dissonant Pythagorean form (pitch 40) is flattened by 1/4th of a comma. Of course this means that our ears hear this interval as a Just Major Sixth (pitch 39) raised by 3/4ths of a comma. The result is tense and beating. We also see that the Just Major Third is 2/3rds of a comma sharp. This is easily enough to impart a certain "restlessness" to the Major Triad. The Minor Thirds and Sixths are correspondingly altered, although they are more tolerable in their "dark" character than the overly "bright" Major Thirds and Sixths.

Thus harmonic patterns like the Pythagorean Tetrad (eg. 0-9-22-31) and the Pythagorean Pentad are well approximated, but the Just Triads and Harmonic Series componants are mediocre. The Seventh Harmonic (ratio 7:4) is so poorly represented that timbres which are rich in the 5th Harmonic and 7th Harmonic beat very badly, and "grate" the ears. This is the reason that accordians and harmoniums tuned into 12-E.T. are so intolerable. For the same reason, vocal ensembles have great difficulty singing in 12-E.T., needing the piano as a reference point and using wide vibrato to "mask" the innaccuracies. Also note that the piano design tries to eliminate the 7th harmonic in its timbre (by judicious placement of the hammers).

Historically the 12-E.T. system was known in theory since ancient times, but was not considered musically useful. The Chinese stuck to pure Pythagoreanism in practice. The ratios were worked out in the early renaissance, but 12-E.T. did not become "universal" in Europe till the mid-19th Century. Various unequal systems dominated the 18th Century, and the 1/4th Comma Meantone system held the longest sway (from the beginning of the 15th Century(?) to the end of the 18th). The exception lay in the simpler fretted instruments and folk instruments, which were set up in some variant of 12 since the middle-ages. But some fretted instruments, eg. lutes, had moveable frets and employed variable intonation.

Another aspect of the character of 12-E.T. should be considered. Remember that the pure Pythagorean Chromatic scale contained within it a polarity between the Pythagorean Ogdoad and the Ptolemaic Ogdoad. The 12 system "homogenizes" this polarity in favour of the Pythagorean sound. This is truly a loss. The resulting effect of the system is definitely restless. It is interesting that the evolution of musical style in 12 has tended toward increased speed and "flash".

Another reason for this restlessness (apart from the tempering itself) is the system's excessive symmetry. Of all the good cyclical temperaments, 12 is the only one which is COMPOSITE. 12 is divisible by 2,3,4, and 6. A relatively large number of its patterns, as a result, are "reductionist". The simplest example is

Greeks were aware of the notion of tempering--an idea which Aristotle called a "relaxed proportion," (NICOMACHEAN ETHICS 1173a). Aristoxenus observed that intervals have an "appreciable locus of variation" (HARMONICS). Intervals of less than one-sixth of a tone are "not harmonic elements" to Aristoxenus. This amounts to about 33 Cents. Remember that the 31 system divides the tone into five parts. Aristoxenus seems here to be endorsing the 31 system. The sacredness of the Golden Mean ratio to the Pythagoreans also implies some connection to this tuning system, as we shall see latter.

To return to our examination of 31-E.T. structure, the vertical nature of the tuning and the presence of the Meantone Corridor naturally lead to the view that the system embodies eight Regions within its Functional Field. These regions are shown on Figure 28. The Major Region and Minor Region will be familiar since these two regions are also present in the 12-E.T. system. But the other regions may not be so familiar. It is interesting that the Chromatic scale already spans three regions. Note that at the top and bottom of the Field is the Enharmonic Region, which joins the whole Field together in a "strange loop." We could say that this "vertical regionality" is also present in Just Intonation and 53-E.T., but it is emphasized in 31-E.T. since the comma-alterations are eliminated, leaving this verticality as the only regional structure.

We can look at Figure 28 as the Field of Functional Triads. For example, 1-3-5 is the Tonic Major Triad, and 6-1-3 is the Relative Minor, and so on. When looked at this way, the triad b7-2-4 and its relative Minor 5-b7-2 "straddle" the Major Region and the Minor Region. In the same way, the triad 2-#4-6 and its relative minor 7-2-#4 straddle the Major Region and the Supra-Major Region. These special triads are called BOUNDARY TRIADS and lie "between" the regions, or somehow within both regions. On the other hand, a triad such as b6-1-b3 is not a Boundary Triad; it falls squarely within the Minor Region. Note that the Boundary Triads also embody maximum tempering within the system since they mostly lie within the Meantone Corridor.

Figure 30 shows the Circle of Fifths in a pairing of the Major Triads and their Relative Minor Triads. The eight regions are highlighted. The triad pairs associated with the regional boundaries are marked with asterisks. We can see that the Enharmonic Region lies at the opposite end to the Major Region. There is a symmetry about the Tonic in the overall regional structure of the system.

I had said above that all cyclical temperaments embodied a functional ambiguity somewhere in its structure. In 31-E.T. the ambiguity lies within the Meantone Corridor, with its associated Boundary Triads. It also lies within the Enharmonic Region which is associated with the Harmonic Antipodes. There is no ambiguity within the Harmonic Heptad and associated structures. An examination of the Functional Field shows us that the "weak"

points in the system all lie in the best places, and not in the central triadic harmonies (as is the case in 12-E.T.).

31-E.T. is a PRIME SYSTEM, whereas 12-E.T. is a COMPOSITE SYSTEM. This prime character asures that the tuning does not have excessive rotational symmetry. Hence there is far less functional ambiguity than in 12-E.T. An example of this "functional integrity" is shown on Figure 36. This diagram shows the structure of the Diminished Tetrad in 31-E.T. Compare it with the Diminished Tetrad of 12-E.T. shown on Figure 24. The natural resolution of this harmony is shown on the diagram. The Tetrad forms a "boundary" to the essential core of the chromatic scale. The resolution is "positioned" by the placement of the Sub-minor Third Interval (here between Ab-B). Thus the pitches B-D-F-Ab naturally resolve to C Major or Minor, ie. "Key of C." In order to produce a natural resolution to the Key of A, for instance, we must use pitches B-D-F-G#, as shown. There is no ambiguity between G# and Ab. Also note that there is only one axis of symmetry. Looking now at the pattern in 12-E.T., we see multiple axes of symmetry and the elimination of the sub-minor third in the mix. As a result, the pattern can resolve equally well to the Key of C, A, F#, or Eb. The nature of the function can only depend on the surrounding harmonic context. Hence the potential for "atonality," which can also be interpreted as "functional confusion." The 31-E.T. system does not embody this confusion in its functional relationships. Hence it is not as well suited for "atonality" but rather for extended forms of Tonality. These new regions of "modal tonality" are as yet barely explored. One of the most intriguing aspects of this system, besides the improvement of conventional harmony, is the exploration of these new harmonic possibilities.

Figure 32 is self-explanatory. It deals with certain aspects of the mediaeval notation and the evolution of the mediaeval Pythagorean chromatic scale into the standard Meantone scale. The need to access different "corners" of the Circle of Fifths led to the notion of a VARIABLE MEANTONE TUNING, since the sharps could be retuned as flats, or vice-versa. Baroque musicians were adept at retuning their harpsichords, since there was no professional class of tuners. The instrument is easily retuned. Bach is said to have been able to retune his entire harpsichord in only 15 minutes. Usually only one or two notes per octave needed retuning, so that a retuning could be accomplished very quickly. All of these tunings were subsets of the 31 system. In the 18th Century there was a lot of tuning experimentation. Some of the other systems treated latter in this article were tried out. But the Quarter-Comma Meantone system was always considered the standard from which one "deviated."

The first composer-theorist to explicitly try to put all 31 tones per octave on a keyboard was Vicentino in the mid 16th Century. The instrument was his famous Archichembalo. But it was too cumbersome to be practical. Probably many of the 18th Century musicians who used a Variable Meantone Tuning had no idea that the line of Fifths was actually a circle which closed after 31 Fifths. The technological problem of their (and our) keyboard design led

eventually to the adoption of an inferior tuning system which suited the keyboard design at hand. A major factor in the adoption of 12-E.T. came when the piano was fitted with the cast metal frame. As a consequence, the tension on the strings could be increased dramatically. However, this increased tension made tuning a much more difficult task. Hence a class of professional piano tuners evolved during the 19th Century who specialized in tuning problems. The piano was no longer amenable to experimentation, and 12-E.T. became the standardized norm. The popularity of the piano and the ability of the other instruments to adjust their intonation led to 12-E.T. dominating the field.

However, many aspects of the Meantone era have been preserved, most notably musical notation. A rational notation for 12-E.T. would do away with the symbols for sharps and flats. But we have preserved this distinction, probably because the musical ear still wants to make that very real distinction in pitch. Orchestras and singers constantly make such distinctions, even though they have had no explicit theoretical training in Meantone tuning. The exclusive use of the piano for the study of harmony has obscured many issues relating to intonation. Fortunately this lack and bias in the educational system is now being remedied. The growth of electronic music is making issues relating to intonation more "open ended," so that we are in some ways moving to a situation analogous to the Baroque era. We suit our tuning to our musical purposes, and not the other way around (ie. limiting our musical purposes to some unquestioned, dictatorial authority).

This particular meantone system presents such fascinating structural complexity that many more topics relating to syntax and mode could be raised. However, these topics are beyond the scope of this introduction. I will therefore conclude this section with a brief reference to Figure 38 and Figure 39.

These two diagrams show the interesting association of this system with the GOLDEN MEAN RATIO, the FIBONACCI SERIES, and the SQUARE ROOT OF FIVE. The Golden Mean is found ubiquitously throughout nature and mathematics. Even though the ratio is not found directly in this system, it is found indirectly as shown. The square root of five is closely associated with the ratio. The whole field of Quarter-comma Meantone can be ideally pictured as consisting of ratios expressible as powers of 5 and 2 (the octave). This tuning system is truly an expression of the Pythagorean "human number" 5.

The Fibonacci Series of temperaments shown embodies four of the "good" systems--12, 19, 31, and 50. Interestingly enough, a continuation of the series in either direction yields nothing worthwhile. I have no idea why this is so. Finally, the three primary axes of Just Intonation, expressed through the ratios 2:3, 3:5, and 5:8, themselves relate to the Fibonacci Series 1,1,2,3,5,8,13...

The association of this system with the Golden Ratio tends to link it to the Pythagorean School, which held the ratio

as sacred. They also esteemed the number 10. Ten is two times five. Five stood for the human world (microcosm), and ten for the cosmos (macrocosm). This number could also be interpreted as refering to the first ten ratios in tuning, ie, 1:2, 2:3, 3:4, 4:5, 5:6, 6:7, 7:8, 8:9, and 9:10. These ratios are all well placed in 31-E.T., except for the "fusion" of 8:9 and 9:10. The number 10 could also refer to the first 10 Harmonics, producing the Harmonic Series Pentad shown on Figure 37. There are other links between the Pythagorean School and this tuning system. Yet there is no explicit reference. Could this be yet another example of Pythagorean secrecy?

Whatever the ancient links, anyone who impartially examines the structure of these various cyclical temperaments would come to the conclusion that the 31 system is, indeed, the best compromise. It offers the greatest harmonic richness, the best Harmonic Series and Sub-harmonic Series approximations, the best blend of symmetrical and complimentary patterns, and the most "singable" triads, without an astronomical number of tones per octave.

We turn now to a very good cyclical temperament which has been known theoretically since the renaissance, but which has never come into wide use.

19-E.T. (QUASI-ONE THIRD COMMA MEANTONE)

This significant tuning system was first proposed by Salinas in the 16th Century. It has had other proponants, especially Yasser in the 20th Century. The system is simple enough that it could be applied to fretted instruments, such as the guitar, an advantage over the 31 system.

In this system the musical Fifths are flattened by 1/3rd of a comma. It is easy to see the logical source of this tempering, since the tuning of three Just Fifths results in a dissonant, comma-raised interval, as shown in Figure 43. The tempering orients the system strongly along the 5:3 Axis of Just Major Sixths and Minor Thirds. Hence the tuning also emphasizes the pitches comprising the ENHARMONIC GATEWAY, which are named D# and A# in this system.

A glance at Figure 41 shows why this is the case. the central axis of the system terminates at pitches whose comma size is 11 and 42. Pitch 11 at the top has a Fifth whose size is 42 minus one-third of a comma. But this pitch is practically identical to pitch 42 at the bottom of the chart and "inside" the boundary. A closed "loop" of Fifths results. We are not surprised to find pitches D# and A# at the opposite side of the circle of Fifths as the generator pitch (C), as shown on Figure 47. These two pitches belong to a special Interval Class unique to this system. Any adequate theory for this system must take the Enharmonic Gateway into account.

From Figure 41 we see that the tempered intervals are altered by either one-third of a comma, or two-thirds of a comma. The first tempering is "borderline" for me, since I can sometimes perceive the innacuracy, especially in sustains. The second tempering is definitely perceptible, being about the same tempering as the 12-E.T. Major Third. This tempering is definitely more "crass" than that of the 31-E.T. system. In other respects also, the 19 system is inferior to the 31 system. Nevertheless, it is still an excellant tuning system.

Let us look at the ten Interval Classes of the system. The first class, Unison and Octave, are perfect as in all systems. The second class, Fifths and Fourths, are altered by 1/3rd comma. The third class, Major Thirds and Minor Sixths, are equally altered. The flat Major Thirds of this system contrast it sharply with the sharp Thirds of 12-E.T. Balbour, (TUNING AND TEMPERAMENT, p. 116) describes them as "insipid in the extreme," but I haven't found them so bad. On the other hand, I find the 1/3rd comma raised Minor Sixth a little more "bizarre." The fourth interval class, Major Sixths and Minor Thirds, are practically perfect. Remember, however, that 19-E.T. is quasi-one third Meantone, the distinction being imperceptible.

In this tuning system we see that the Minor Triad is less tempered than the Major Triad. Whether this situation is of advantage or not, I leave you to decide. The quality of the various chord types are greatly at odds with 12-E.T. and with 31-E.T. This tuning system has great "individuality" and it is definitely ground for new harmonic spaces.

The fifth Interval Class, wholesteps and Minor Sevenths, are also at odds with 12-E.T. For 19-E.T. uses a small wholestep of about eight and one third commas in size. Hence the 9th harmonic is significantly mis-tuned. The Minor Seventh is strongly "Just" and not "pythagorean." We see that there is no representative at all for the 7th Harmonic, since the pitch A# is also nowhere near it. Hence this tuning system is like 12-E.T. in negating the Septimal ratios. In this regard it is definitely inferior to the 31 system.

The sixth Interval Class, diatonic semitones and Major Sevenths, are also unique. For the semitone, normally about five commas in size, is "stretched" by two-thirds of a comma, making it unusually large. The Major Seventh is correspondingly flattened and is one of the least "desireable" intervals of the system. But it is still useable.

The seventh Interval Class, tritones, are quite interesting. A look at Figure 41 shows both the sharpened fourth and flattened fifth on the central line of best intervals as pitches 25 and 28. Here we have maximum differentiation between the two tritones. In fact, if we consider the chromatic sequence 22 (F), 25 (F#), 28 (Gb), 31 (G), we see that the wholestep is divided exactly into three. These metrical properties are easily seen on the Circular Graph which is Figure 46. As a result the

diatonic semitone is twice the size of the small, chromatic semitone, an interval of class eight.

The 12-E.T. system is complete in the above seven Interval Classes, but the 19-E.T. system adds three more "dissonant" classes to make a total of ten. The eighth Interval Class is the small semitone mentioned above and its inversion, the oddly "stretched" Major Seventh, which is really a "flattened octave." The ninth interval class consists of a "stretched" fifth and a correspondingly "squeezed" fourth which we may appropriately call Wolf intervals. This interval subsists between pitches C and G#, for example. Consequently, a very bizarre "Wolf" Major Triad and Minor Triad are also possible in this tuning. For example, the Wolf Major Triad on C has pitches C-E#-G, the Wolf Minor Triad C-D#-G. In addition, an even stranger symmetrical Wolf Triad is possible, using pitches C-E#-G#, for example. Note the position of these three pitches on the Pitch Field. E# and G# "span" the two ends. This strange chord is unique to 19-E.T.

The tenth and last Interval Class gives us the intervals associated with the Enharmonic Gateway. The interval between C and D# can only be described as a "squeezed" minor third or an over-sized wholestep--it really is within the "neutral" territory between the wholestep and the Minor Third. Its inversion, eg. C-A#, lies between a Major Sixth and a sharp Major Sixth, the strangest interval in the whole system. Maybe it is this interval which caused Ivor Darreg to describe 19-E.T. as a system with a lot of "zonk." It certainly has character!

Figure 42 shows the Pitch Field. The notation is straight-forward. Generally only two pitches are written as "enharmonics"--E# and B# which become Fb and Cb. Sometimes it is necessary to consider sharps as double-flats and flats as double-sharps, as shown. The notation presents no problems. Note how the pitches F# and Gb (the tritones) relate to the pitches of the Enharmonic Gateway--D# and A#. They form Major and Minor Triads. It is within this enharmonic region that pitches sometimes must be renamed for clear notation. For example, the Major Triad on D#, D#-Gb-A# is best renamed D#-Fx-A#.

Figure 44 and Figure 45 show the SIX REGIONS associated with this tuning, on the Field Diagram and on the Circle of Fifths. Note that the Enharmonic Region is considerably expanded in comparison with the regions of 31-E.T. In this regard, too, the 31 system is superior. Again we have "boundary triads" which have maximum "deviance." The boundary triads are again paired by major and relative minor. The Field Diagram may also be regionally divided into three, shown by the thick, broken lines. At the upper corner the intervals tend to mimick comma-lowered forms, and at the lower corner the intervals mimick comma-raised forms. This "spreading" of the system into the comma-altered regions of the Just Field is reminiscent of 12-E.T. Once again, 31-E.T. shows itself superior to both these systems.

Figure 48 shows Zarlino's mid-16th Century harpsichord keyboard which he intended for a segment of the One-quarter Comma Meantone tuning. This keyboard would play the entire 19-E.T. cycle. Unfortunately it is awkward to finger. The 19-E.T. system, however, works wonderfully on a Bosanquet Generalized Keyboard. Figure 48 also shows a 19 tone segment of the 1/5th Comma Meantone tuning. It is the system to which we now turn.

43-E.T. (QUASI-ONE FIFTH COMMA MEANTONE)

This admirable intonation system is associated with the name of Sauveur (1701), since he used it as a unit of musical measure, in a way similar to my use of the comma (53) as a significant measure. An argument could be made for any of the above systems, and the 31 system was also used for this purpose by Van Blankenburg (Hague, 1739). Sauveur named the 43-E.T. step interval a MERIDE, and it was further divided into seven, thus producing 43 * 7 = 301-E.T., basically a scale of "double-schismas." This small step he called an EPTAMERIDE. For more subtle distinctions he recommended dividing these by ten, resulting in an octave divided into 3010 DECAMERIDES. Since .30103 is the common logarithm of 2, it made simple conversion from logarithms to steps by simply dropping the appropriate decimal point. The charms of this 43 system, however, extend beyond ease of calculation.

The essential source of this system is shown in Figure 54. The tuning of five pure Fifths from C to /B, results in the pythagorean, comma-raised Major Seventh (49 commas in size). Therefore, flatten each Fifth by 1/5 comma, and the Major Seventh becomes pure (48 commas). We now see that the Major Thirds and the Fifths to have equal but opposite error of about 4.3 Cents. The Major Sixth is raised 2/5 comma, about 8.6 Cents, an amount which I consider perceptible. The other intervals are altered as shown on Figure 49.

The system orients along a line which leaves the Diatonic Semitone (5 Commas) and Major Seventh (48) pure, and lies exactly perpendicular to the axis of the 19-E.T. system. Although it thus appears to "bypass" the Enharmonic Gateway, it neverthless reproduces it at the two "ends" of the field (pitches 11 and 42). It therefore expresses the Enharmonic Gateway as well as the Harmonic Antipodes. It also presents a rich brood of Interval Classes (22 in all, compared to 31's 16), and harmonic structures. I shall only consider the most important intervals here.

The musical Fifths are quite good, since 1/5 comma is negligible. Major thirds are raised an equal amount--also good. The Minor Thirds are flattened 2/5 comma, a sizable amount. Therefore in this system we see a "heightened" contrast between the minor and the major. This contrast increases with the 1/6 Comma system, the 1/7 Comma system, and so on, to reach maximum contrast in 12-E.T. The "character" of the 43 system lies therefore "between" the 1/4 Comma system and the 1/6 Comma system.

In the 43 system, the Major Sixth is noticeably sharp, 2/5 comma, but not so sharp as in the 12 system (where it is 3/4 comma sharp).

The meantone itself is 2/5 comma flat of the 9:8 ratio. The Minor Seventh is similarly altered. They are, as usual in meantone temperaments, quite usable. The semitones are also interesting. The Diatonic Semitone is, of course, pure (in reality quasi-pure) at about five commas in size. The Chromatic Semitone (in just, 3 commas), is raised 3/5 comma. Thus it is best considered as a Pythagorean Semitone (4 commas), lowered by 2/5 comma. A glance at the Circular Graph (Figure 51) shows that the chromatic semitone (C#) is closer to the Diatonic Semitone (Db) than the corresponding pitches in the 31 system, and especially the 19 system. The step difference is now only about 27.9 Cents--an oversized comma (raised by 1/5 comma). We see from the Circular Graph that the chromatic semitone is three steps in size, the diatonic semitone four steps. In this respect as well as others, this system seems to lie "midway" in character on a continuum which has the 19 system at one end, then the 31 system, 43 system, 55 system, and so on, with 12-E.T. at the opposite extreme of the continuum. 43-E.T. is a happy middle ground.

Figure 50 shows the pitch Field with the step number superscript. Looking at pitch A# which is the system approximation of the 7th Harmonic, we see that it is quasi-just, and therefore sharp (remember 53-E.T.?). In fact, it is 7.9 Cents sharp, not really very good. Pitch D# which approximates the 7:6 ratio is even worse, being about 12.2 Cents sharp. In this respect this tuning system is also inferior to the 31-E.T. system. In fact, all the systems are inferior to the 31 system here.

The regional structure of the Functional Field is shown on Figure 52, and Figure 53. It is quite straightforward, and reminds us of the 31-E.T. structure, with boundary triads meeting in major-minor pairs. Note here the small extent of the enharmonic region--where the two ends meet to form a closed "loop." As usual, these pitches lie opposite the generator pitch. In fact, these pitches (step numbers 9 and 34) are representatives of the Enharmonic Gateway "reborn" so to speak.

The Circle of Fifths is also featured on Figure 55. The notation of this system, unfortunately involves more complexity than that for 31 or 19. Here is another point against the 43 system. Still, it is workable. We shall see, however, that this is the last system of this series which we can consider workable, once we weigh in balance the various pros and cons. We shall see that the 1/3 Comma system, 1/4 Comma system, and the 1/5 Comma system form the best of the lot. They combine with the 12 system and the 53 system to make the five best tuning systems. Of these five, the 53 and 31 systems are undoubtedly best.

55-E.T. (QUASI-ONE SIXTH COMMA MEANTONE) and beyond!

Figure 56 shows the Field for the 55-E.T. system. Here we have the first of the meantone series in which the complexity does not seem worth it when one examines the character of the tempering. We now have a tuning in which the Fifths and Fourths are very good, but at the expense of the Thirds and Sixths. Major Thirds are 1/3 comma sharp--borderline, but Major Sixths are 1/2 comma sharp--quite noticeable. Minor Thirds are 1/2 comma flat, producing a marked contrast between minor and major intervals. Thus the character of the tuning system approaches that of 12-E.T. The advantages of 12-E.T. over 55-E.T. however, are obvious because of the latter's complexity. The notation becomes more difficult, for one thing.

55-E.T. is essentially a scale of syntonic commas. But if we are going to put up with such a complex scale, why not use the 53-E.T. scale, since it is so superior an approximation of Just Intonation? Here is another argument against its utility.

This tuning, nevertheless, has seen a lot of use. For during the 18th Century a chromatic scale using 1/6 Comma tempering was quite popular. The principal reason was probably that the "Wolf" fifth interval for this system is not very large. Hence it was proposed over the 31 system for the standard keyboard. We now know that the 12 system is probably the best tuning for that particular keyboard design. The German Silbermann promoted the 1/6 Comma tuning, and no doubt it was known by Bach. The composer Telemann favoured the 55 division, and the chromatic segment from Eb to G# was used by a wide segment of the 18th Century musicians.

The 1/6 Comma system is also historically interesting because of its alteration to produce a number of important "Well Temperaments" associated with Neidhart, Marpurg, and Young. Using a combination of 1/6th comma temperings, and 1/12th comma temperings (remember the schisma?), these theorists produced a number of irregular chromatic scales with special characteristics. In general, they took the inherent polarity of the Pythagorean Chromatic scale, which exists between the Pythagorean Ogdoad, and the Ptolemaic Ogdoad, and distributed it around the 12 keys. The result is a "circulating temperament" in which there is a steady change of character between the two extremes in the various keys. Each key has a different character, with the key of C usually closest to just, and F# closest to pythagorean. Unlike a chromatic segment in 1/4 comma tuning where certain keys are quite dissonant, here all keys are usuable. It was for one of these circulating temperaments that Bach wrote his Well-Tempered Klavier, not for 12-E.T. as was formerly thought. There is no doubt, though, that all this experimentation with many irregular tunings, which all somewhat approximate 12-E.T., must have broke ground for the acceptance of the 12-E.T. system.

The special properties of the division of the comma into 6 and 12 parts no doubt derive from the schisma. All of the "circulating" temperaments are best thought of as subsets of the scale of schismas, 612-E.T. We shall not treat these

Well-temperaments any further here since this article concentrates on the cyclical meantone temperaments. Rather, let us continue the series.

Figures 57 through 61 describe the fields for 1/7 Comma, 1/8 Comma, and so on to 1/11 Comma Meantone Temperament. The problems which first arose in connection with the 1/6 Comma system are here greatly amplified. For the complexity of the division quickly becomes astronomical, and approaches the schisma scale. Remember that 1/11 syntonic comma is basically a schisma. On top of this complexity, the quality of the scale becomes indistinguishable from the 12-E.T. scale. Consequently, there is little point to using such an extreme division. If the 1/6 Comma system was borderline, these systems are beyond the bounds of the practical.

We could extend the series the other way, producing the 1/2 Comma system, 26-E.T., shown on Figure 62. Here the meantone is at its extreme smallness in size, 8 commas. But note that the Major Third is a whole comma flat (at 16 commas). This system is musically bizarre and not worth considering. The 1/3 Comma system (19-E.T.) was sometimes called a "negative meantone" tuning because the Major Third is flattened rather than sharpened as in most other temperaments. If that is the case, then this 1/2 Comma system is the most "negative" of the whole series.

Thus we come to the end of this series. We could, however, consider another series in which the numerator of the fraction is two rather than one. Here we get 2/5, 2/7, 2/9, and 2/11 Comma Meantone, as shown on Figures 63 through 66. But the only system worth considering is the 50-E.T. system, 2/7 Comma Meantone, shown on Figure 64. This system was introduced by Zarlino during the renaissance. It is interesting in that it presents a character mid-way between the 19 and the 31 system. Thus it is only mildly "negative." It is also the most extremely "vertical" of all the temperaments, as can be seen on the diagram. Yet it suffers from a similar problem as does 55-E.T., namely its proximity in complexity to the 53-E.T. system. Why would we chose this tuning over the Quasi-Just (53) tuning? Whereas the 55 system is a scale of syntonic commas, the 50 system is a scale of ditonic commas. Note its position in the fibonacci series of temperaments--12, 19, 31, and 50.

The 74-E.T. system is musically interesting, but too complex. On the other hand, the 45-E.T. system is less complex but excessively "negative." Thus none of the temperaments in this series is "first class." We have now definitely reached the end of the road. Therefore we must conclude that the best quasi-meantone temperaments are the 12, 19, 31, 43, and 53 systems.

Figure 67 compares the division of the wholestep in each of these major systems. Figure 68 shows the size of the characteristic "Wolf" interval within the major meantone systems. We see a gradual reduction in the size of the interval as we approach the 1/12 Comma tuning. The 53 system is not shown since

it "stands outside" the series proper. In many ways the 12 system and the 53 system form a special class separate from the others in the "big five." If we use a colour analogy, 12 and 53 are black and white, whereas the other three are the primaries, red, yellow, and blue. These five systems also remind one of the five regular polyhedra, shapes sacred to the Pythagorean School of philosophy. It is also interesting that the three "coloured" systems, 1/3, 1/4, and 1/5 Comma Meantone, relate by 3,4,5--the traditional Pythagorean Triangle (also the Just Major Triad in harmonics). Here we have another expression of "fiveness."

Figure 69 outlines the Interval Classes of the big five. I have used the order of the cycle of Fifths in their numbering. We would need to rearrange the order if we numbered them according to the continuum of consonance to dissonance. Such an ordering is important in understanding the syntax of structural harmony for each system.

Figures 70 to 73 show the application of these tunings to the Bosanquet Generalized Keyboard, which make these tunings practical and easy to play. Ironically, the 53 system is absent, although the inventor intended it only for 53. The keyboard is pictured in its original form on page 392 of Partch's book: GENESIS OF A MUSIC. On building a cardboard mock-up in order to feel it under my fingers, I found it awkward to finger when playing Just or Septimal patterns. It was only comfortable for Pythagorean patterns. A practical keyboard for 53-E.T. has yet to be developed. I have worked out a design which I feel has promise.

But for the other systems the Bosanquet keyboard is excellant, including the 12 system. It is a vastly superior keyboard for 12 than the old, piano keyboard. Through the medium of electronics this keyboard could instantly switch between any of these four, without the need to relearn fingering patterns.

I conclude this introduction to the Meantone cyclical temperaments with a short observation on divisions "in general." For one can divide the octave into any number of equal-tempered tones, and pay no attention to the meantone series at all. This approach has been systematically explored by a number of theorists since the 19th Century. Donald E. Hall in the article: A SYSTEMATIC EVALUATION OF EQUAL TEMPERAMENTS THROUGH N = 612 (Interface, Vol.14, 1985), reviews the literature and presents an objective measure of value. His findings generally concur with mine, although I have not been considering very complex systems like 441 or 342. Interestingly enough, he finds the 41 system superior to the 43. Also, the 72 system is superior to the 74. It is fascinating how the "good" divisions seem to group in "clumps," witness 50, 53, and 55. The 22 division is also worth considering. He concurs with Mandelbaum that the 19, 31, and 53 divisions offer "distinct acoustical refinements" over the others.

In my experience, the 31 system is the best one, due to the excellant 7th harmonic ratios, neutral tones, and just tones. The only rival is the 53 system, which involves considerably

increased complexity. The invention of such a good keyboard for 31, as well as its historical importance, make it the most likely candidate for the future evolution of harmony. The last century has been dominated by the 12 system, but the future will probably see the "peaceful co-existence" of the "big five."

BIBLIOGRAPHY OF LITERATURE CITED

Barbour, J.M.: TUNING AND TEMPERAMENT, A HISTORICAL SURVEY; East Lansing, 1951.

Bosanquet, R.H.M.: AN ELEMENTARY TREATISE ON MUSICAL INTERVALS AND TEMPERAMENT; London, 1876.

Hall, Donald E.: A SYSTEMATIC EVALUATION OF EQUAL TEMPERAMENTS THROUGH N-612; Interface, Vol.14; Utrecht, 1985

Helmholtz, H.L.F.: SENSATIONS OF TONE; London, 1885.

Salinas, F.: DE MUSICA, LIBRIS VII; Salamanca, 1577.

Terpstra, S.: REFLECTIONS ON AN IMPROVED NOTATION SYSTEM FOR 53-E.T. AND JUST INTONATION; Interface, Vol. 14; Utrecht, 1985.

Vicentino, N.: L'ANTICA MUSICA RIDOTTA ALLA MODERNA PRATTICA; Rome, 1555.

Zarlino, G.: DIMONSTRATIONI ARMONICHE; Venice, 1571.