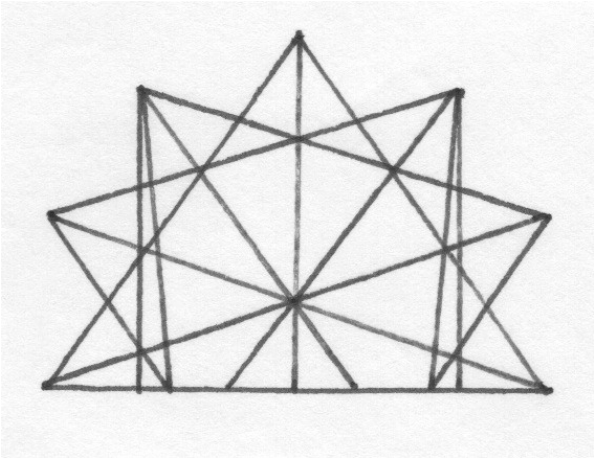
An Abbreviated Dictionary of 31-et Harmonies



Siemen Terpstra

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theoretical language to work just as well for Monteverdi as for modern 'experiments.' I find Fokker's collection of sets a bit 'abstract' and not so relevant.

Of course, both Fokker or myself cannot hope but give only a small sampling of the possible patterns in the 31-et system. There are 2^{31} (that is, 2,147,483,615) sets in the system. Now if we 'clump' these sets into types,(eg. the major triad is one 'type' or class, of which there are 31 members) then there are $2^{31}/31$ or 69,273,665 types. In this dictionary, I have presented only a paltry 260 (or so) types. But I would claim that these are the most musically useful patterns, both historically, and in an extended-modal style which is natural to 31-et. I will let the reader judge whether the collection of sets presented by Fokker includes the most musically useful harmonies.

Most of the harmonies herein are subsets of the Chromatic Dodecad. Many are subsets of the Diatonic Ogdoad and Diatonic Heptad. There are many more triads, tetrads, and heptads than there are pentads, or hexads, since these are the harmonies most referred to in classical theory. A few of these harmonies were discovered on the guitar, but most come straight out of the logic of the matrix or else from my analyses of the renaissance-baroque literature. Every so often, I add another harmony to the group. In some ways, this collection will never be 'finished,' but that is of no consequence. All of the traditional harmonies and many extensions of them are included. Depending upon one's musical style, one is liable to find it here. If not, chances are it is something quite exotic! Of course, this is what the composer may want. This tuning system offers a vast array of possibilities. Nevertheless, it seems to me that the traditional harmonies serve as a good foundation.

I have given only the *primary functions*, that is, functions that contain the tonic. Obviously, any pattern has 31 possible functions, but they can be located instantly using the HARMONY FINDER. I have used a functional notation rather than the pitch notation, since it is more relevant to the understanding of relatedness in the system. Anyway, function and pitch can be instantly translated into each other using the HARMONY FINDER.

The reader familiar with the collection presented in my paper *Notes,* will notice a small group of sets that were not included there. These sets have an 'Arabic' quality of sound, and have special properties. They are harmonies which use the 'end zones' or 'harmonic antipodes' in their structure. These patterns are not found in classical theory (or Fokker's sets). I found these scales useful since I have been playing guitar with some musicians from the middle-east. These scales give me 'quarter-tones' and other exotica useful for this musical style. Hence I have included them in the collection. However, I make no claims to have exhausted the musically interesting scales in this wonderful tuning system. This collection of harmonies cannot help but be ideosyncratic, but I hope that it does include all that is necessary for an intelligent analysis of the past literature and also for new composition.

Figure 1: A comparison between the Fokker and Terpstra approach in 'mapping' harmonies on the Matrix. Only a few examples are given.

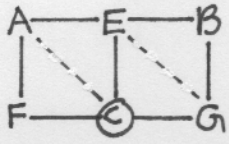
Fokker	Terpstra

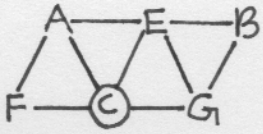
The Suspended Tetrad

Genus {333}

The Ptolemaic Hexad

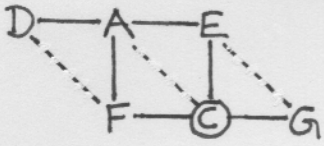
{335}





The Hexachord

{?}



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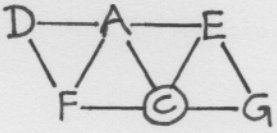
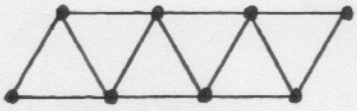
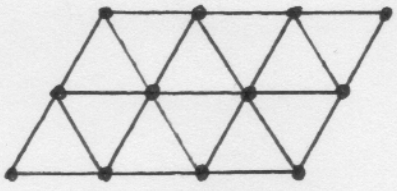


Figure 2: Most harmonies herein are subsets of a few significant patterns:

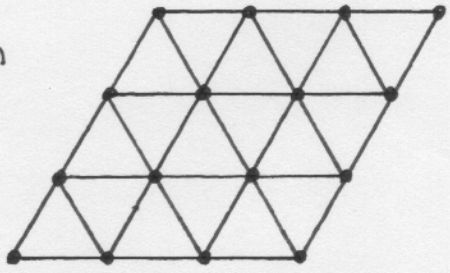
The Full Diatonic Ogdoad



The Chromatic Dodecad



The Full Enharmonic pattern



The Intervals

The complete 16 Classes

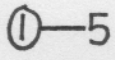
[0] - Unison

[31] - Octave

Diatonic Genus:

[18] - Fifth (5)

[13] - Fourth (4)

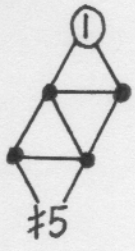


No Chromatic Genus

Enharmonic Genus:

[19] - "Wolf" Fifth
Double-Flat Sixth (bb6)
Semi-sharp Fifth (♯5)

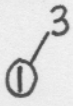
[12] - "Wolf" Fourth
Sharp Third (#3)
Semi-Flat Fourth (♭4)



Diatonic Genus:

[10] - Major Third (3)

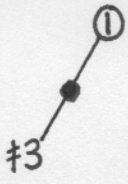
[21] - Minor Sixth (b6)
Sesqui-sharp Fifth (#5)



Chromatic Genus:

[11] - Supra-Major Third
Semi-sharp Third (♯3)
Flat Fourth (b4)

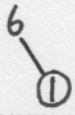
[20] - Sub-minor Sixth
Sharp Fifth (#5)
Sesqui-Flat Sixth (♭6)



Diatonic Genus:

[23] - Major Sixth (6)

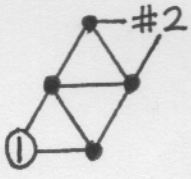
[8] - Minor Third (b3)
Sesqui-sharp Second (#2)



Chromatic Genus:

[24] - Supra-Major Sixth
Semi-sharp Sixth (♯6)
Double-Flat Seventh (bb7)

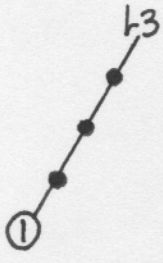
[7] - Sub-Minor Third
Sharp Second (#2)
Sesqui-Flat Third (♭3)

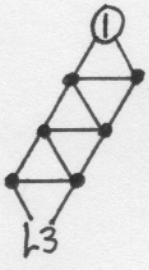


Enharmonic Genus:

[9] - Neutral Third
Semi-flat Third (♭3)
Double-sharp Second (x2)

[22] - Neutral Sixth
Semi-Flat Sixth (♭6)
Double-sharp Fifth (x5)

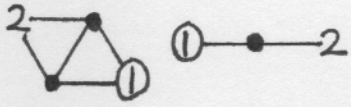




Diatonic Genus:

5 - Wholetone (2)

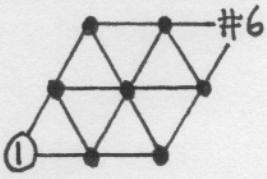
26 - Minor Seventh
Flat Seventh (b7)
Sesqui-sharp Sixth (#6)



Chromatic Genus:

6 - Septimal Wholetone
Semi-sharp Second (♯2)
Double-flat Third (bb3)

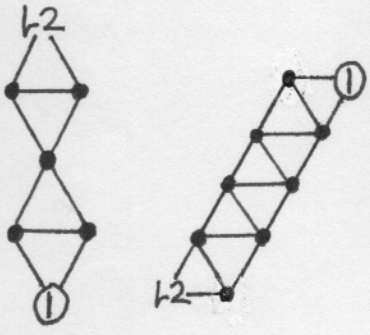
25 - Septimal Minor Seventh
Sharp Sixth (#6)
Sesqui-flat Seventh (♭7)



Enharmonic Genus:

4 - Semi-flat Second (♭2)
Double-sharp Unison (×1)

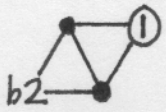
27 - Neutral Seventh
Semi-flat Seventh (♭7)
Double-sharp Sixth (×6)



Diatonic Genus:

3 - Diatonic Semitone
Flat Second (b2)
Sesqui-sharp Unison (#1)

28 - Major Seventh (7)



Chromatic Genus:

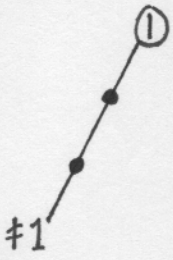
2 - Chromatic Semitone
Sharp Unison (#1)
Sesqui-flat Second (♭2)

29 - Flat Octave (b1)
Semi-sharp Seventh (♯7)

Enharmonic Genus:

1 - Diesis, enharmonic
Semi-sharp Unison (♯1)
Double-flat Second (bb2)

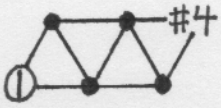
30 - Semi-flat Octave (♭1)
Sharp Seventh (#7)



Diatonic Genus:

15 - Augmented Fourth
Sharp Fourth (#4)
Sesqui-flat Fifth (♭5)

16 - Diminished Fifth
Flat Fifth (b5)
Sesqui-sharp Fourth (#4)

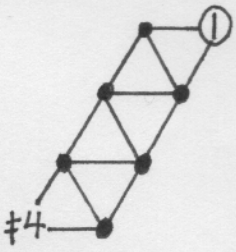


No Chromatic Genus

Enharmonic Genus:

14 - Semi-sharp Fourth (♯4)
Double-flat Fifth (bb5)

17 - Semi-flat Fifth (♭5)
Double-sharp Fourth (×4)

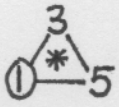


The Triads

A sampling of musically useful 31-E.T. Triad harmonies. A sample matrix pattern and the primary functions are given. Patterns which have modal complimentarity (mirror imaging) are juxtaposed; symmetrical harmonies are centred. Where there are versions from different genera, the usual order Diatonic, Chromatic and Enharmonic is maintained.

Major Triad (MΔ)

1	3	5	– IMΔ
b6	1	b3	– bVIMΔ
4	6	1	– IVMΔ

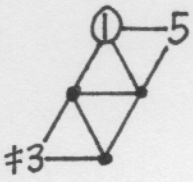


Minor Triad (mΔ)

1	b3	5	– ImΔ
6	1	3	– VImΔ
4	b6	1	– IVmΔ

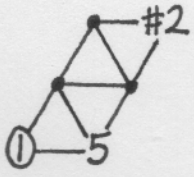
Supra-Major Δ / Septimal Major Δ

1	♯3	5	–
#5	1	#2	–
4	♯6	1	–



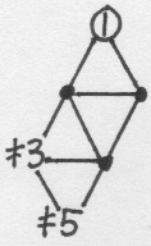
Sub-Minor Δ / Septimal Minor Δ

1	#2	5	–
♯6	1	♯3	–
4	#5	1	–



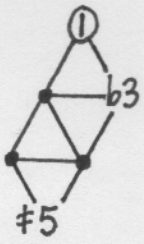
'Wolf' Major Δ

1	♯3	♯5	–
#5	1	b3	–
b4	6	1	–



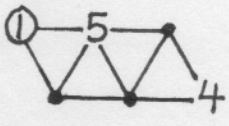
'Wolf' Minor Δ

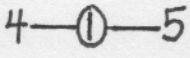
1	b3	♯5	–
6	1	♯3	–
#3	#5	1	–

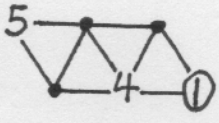


Suspended Δ (SusΔ) – Symmetrical

1	4	5	– Is4Δ
5	1	2	– Is2Δ
4	b7	1	–

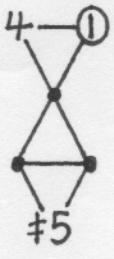






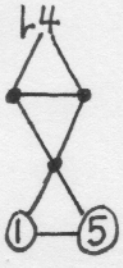
Septimal Suspended Δ ①

1	4	♯5	–
5	1	♯2	–
#3	#6	1	–



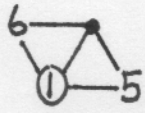
Septimal Suspended Δ ②

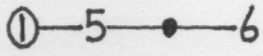
1	b4	5	–
♯5	1	♯2	–
4	#6	1	–

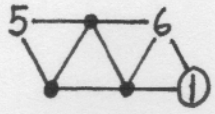


The Six Triad (6Δ)

1 5 6 - I6Δ
4 1 2 -
b3 b7 1 - Im7Δ

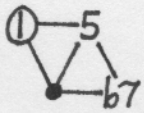


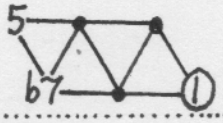




The Seven Triad (7Δ)

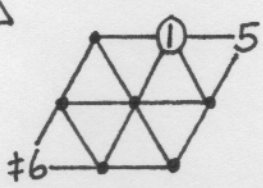
1 5 b7 - I7Δ
4 1 b3 - Im+4Δ
2 6 1 -





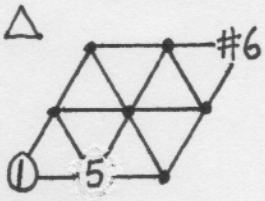
Septimal Six Δ

1 5 ♯6 -
4 1 ♯2 -
#2 #6 1 -

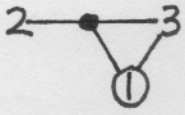


Septimal Seven Δ

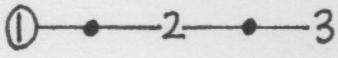
1 5 #6 -
4 1 #2 -
♯2 ♯6 1 -



The Nine Δ (9Δ) - symmetrical

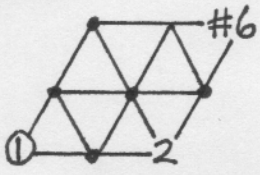


1 2 3 - I9Δ
b7 1 2 -
b6 b7 1 -



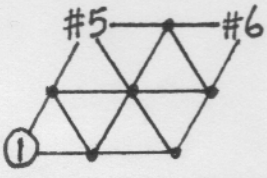
Septimal Nine Δ ①

1 ♯2 ♯3 -
#6 1 2 -
#5 b7 1 -



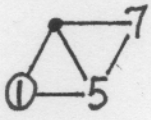
Septimal Nine Δ ②

1 2 ♯3 -
b7 1 ♯2 -
#5 #6 1 -



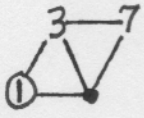
Major Seven Δ (M7Δ)

1 5 7 - IM7Δ
4 1 3 - IM+4Δ
b2 b6 1 -



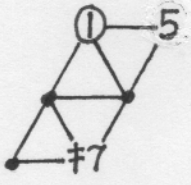
Minor Sixth Δ (m6Δ)

1 5 b6 - Im6Δ
4 1 b2 -
3 7 1 - IM$^{7}_{3}$Δ



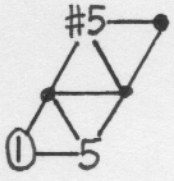
Septimal Major Seven Δ

1 5 ♯7 -
4 1 ♯3 -
#1 #5 1 -



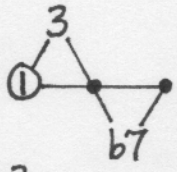
Septimal Minor Sixth Δ

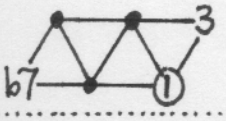
1 5 #5 -
4 1 #1 -
b4 b1 1 -



Dominant Seven Δ (^{7}Δ)

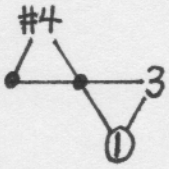
1 3 b7 – I ^{7}Δ
b6 1 b5 –
2 #4 1 –





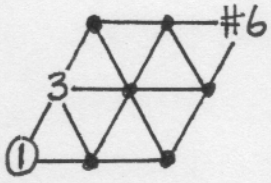
Major Flat-Five Δ (Mb5Δ)

1 3 #4 – I Mb5Δ
b6 1 2 – "IV m6Δ"
b5 b7 1 –



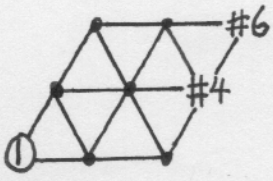
Septimal Dominant Seven Δ

1 3 #6 –
b6 1 #4 –
bb3 b5 1 –



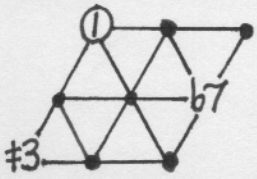
Septimal Major Flat-Five Δ

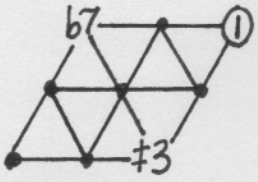
1 3 b5 –
b6 1 bb3 –
#4 #6 1 –



Dissonant Dominant Seven Δ

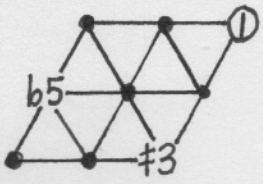
1 ♯3 b7 –
#5 1 #4 –
2 b5 1 –

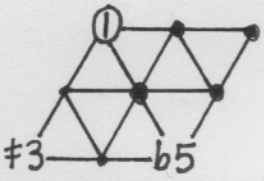




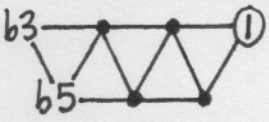
Dissonant Major Flat-Five Δ

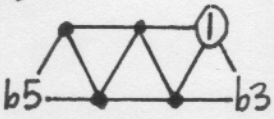
1 ♯3 b5 –
#5 1 2 –
#4 b7 1 –



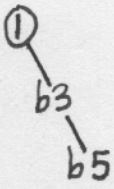


Diminished Triad (°Δ) – Symmetrical



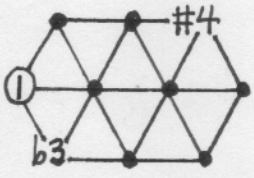


1 b3 6 – VI °Δ
6 1 #4 – #IV °Δ
b3 b5 1 – I °Δ



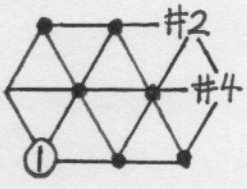
Septimal °Δ ①

1 #2 6 –
♯6 1 b5 –
b3 #4 1 –



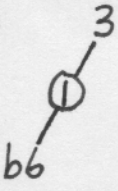
Septimal °Δ ②

1 b3 ♯6 –
6 1 b5 –
#2 #4 1 –



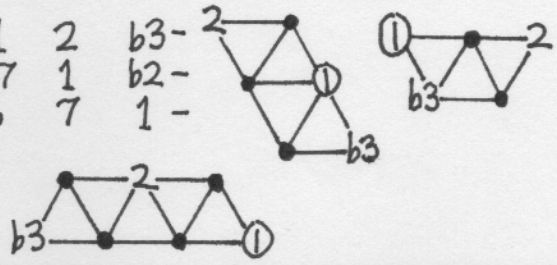
Augmented Triad (+Δ) – symmetrical

1 3 b6 – b VI +Δ
b6 1 b4 –
3 #5 1 – I +Δ



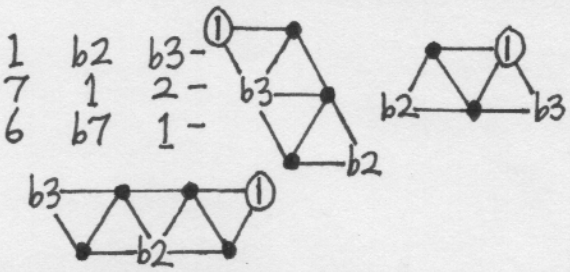
Minor Nine Scalar Δ (m9Δ)

1	2	b3 -
b7	1	b2 -
6	7	1 -



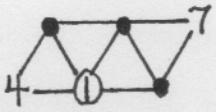
Minor Flat-nine Scalar Δ (mb9)

1	b2	b3 -
7	1	2 -
6	b7	1 -



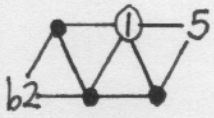
Five Flat-Five Δ (5b5Δ)

1	#4	5 -
b5	1	b2 -
4	7	1 -



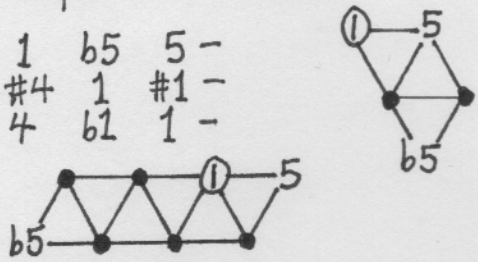
Five Flat-nine Δ (5b9Δ)

1	b2	5 -
7	1	#4 -
4	b5	1 -



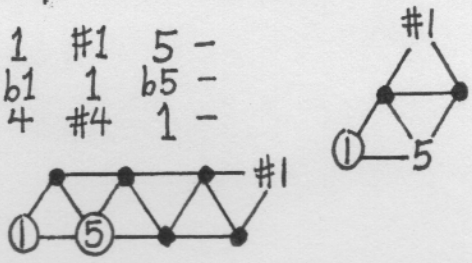
Septimal 5b5Δ

1	b5	5 -
#4	1	#1 -
4	b1	1 -



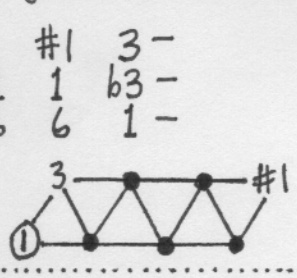
Septimal 5b9Δ

1	#1	5 -
b1	1	b5 -
4	#4	1 -



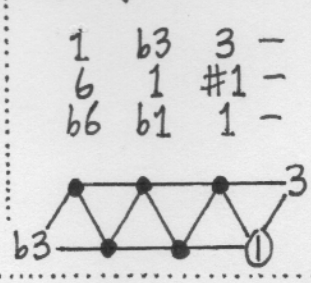
Major Flat-nine Δ (Mb9Δ)

1	#1	3 -
b1	1	b3 -
b6	6	1 -



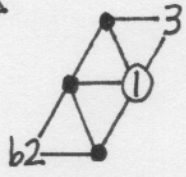
Major-Minor Δ (MmΔ)

1	b3	3 -
6	1	#1 -
b6	b1	1 -



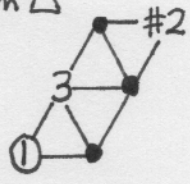
Septimal Mb9Δ

1	b2	3 -
7	1	#2 -
b6	bb7	1 -



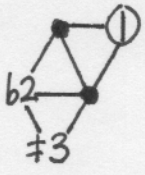
Septimal MmΔ

1	#2	3 -
bb7	1	b2 -
b6	7	1 -



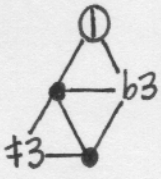
Dissonant Mb9Δ

1	b2	♯3 -
7	1	b3 - m47Δ
#5	6	1 -



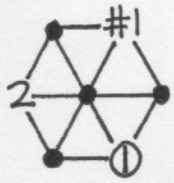
Dissonant MmΔ

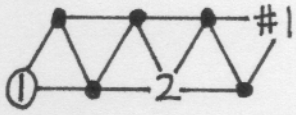
1	b3	♯3 -
6	1	b2 -
#5	7	1 -



Chromatic Δ ①

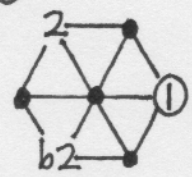
1 #1 2 –
b1 1 b2 –
b7 7 1 –

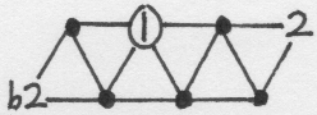




Chromatic Δ ②

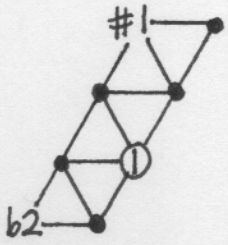
1 b2 2 –
7 1 #1 –
b7 b1 1 –





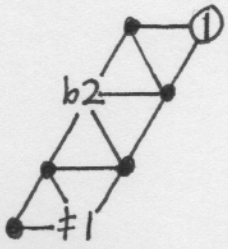
Enharmonic Δ ①

1 #1 b2 –
b1 1 #1 –
7 b1 1 –



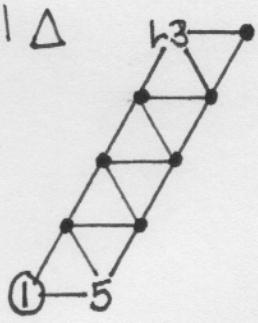
Enharmonic Δ ②

1 #1 b2 –
b1 1 #1 –
7 b1 1 –



A very 'distant' Δ : The Neutral Triad (Symmetrical)

1 b3 5 – Neutral Δ
b6 1 b3 –
4 b6 1 –

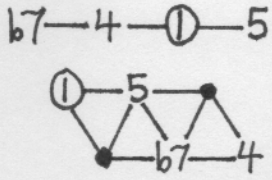


The Tetrads:

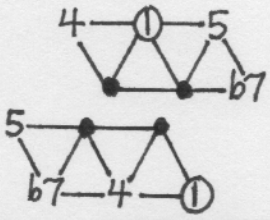
This set of patterns is divided into two groups: the Chordal Tetrads and the Scalar Tetrads. The second group is also designated Tetrachords. All patterns can be seen both as chords and scales, depending on simultaneous or consecutive realization.

Chordal Tetrads:

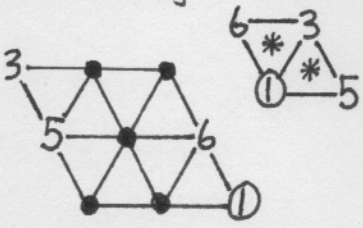
The Suspended Tetrad (Sus □) – Symmetrical



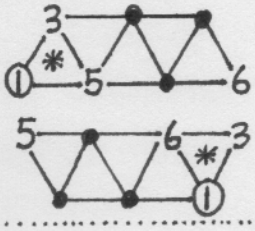
1	4	5	b7	– I 7s4
5	1	2	4	– I s$\frac{4}{2}$
4	b7	1	b3	–
2	5	6	1	–



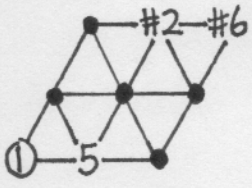
The Major Sixth – Minor Seventh □ (M6□, m7□) – Symmetrical



1	3	5	6	– M6□
b6	1	b3	4	–
4	6	1	2	–
b3	5	b7	1	– m7□

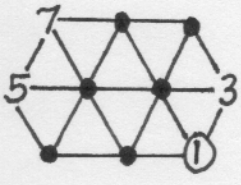


Septimal Major-Sixth, Minor-Seventh □ – symmetrical

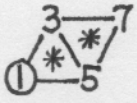


1	♯3	5	♯6	– "M6"
#5	1	#2	4	–
4	♯6	1	♯2	–
#2	5	#6	1	– "m7"

Major-Seventh, Minor-Flat-Six □ (M7□ – mb6□) – Symmetrical

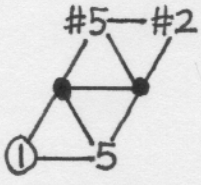


1	3	5	7	– IM7□
b6	1	b3	5	– Imb6□
4	6	1	3	–
b2	4	b6	1	–



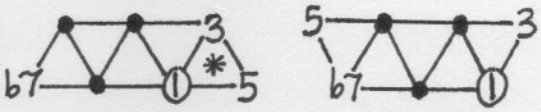
Septimal Major-Seventh, Minor-Flat-Six □ – Symmetrical

1	♯3	5	♯7	– M7
#5	1	#2	5	– mb6
4	♯6	1	♯3	–
#1	4	#5	1	–



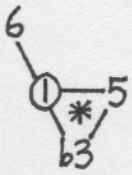
Dominant Seven Tetrad (Dom7□)

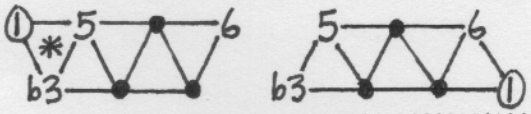
1	3	5	b7 – I⁷□
b6	1	b3	b5 –
4	6	1	b7 –
2	#4	6	1 –



Minor Six Tetrad (m6□)

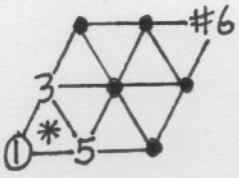
1	b3	5	6 – Im6□
6	1	3	#4 –
4	b6	1	2 – IVm6
b3	b5	b7	1 – Iø7





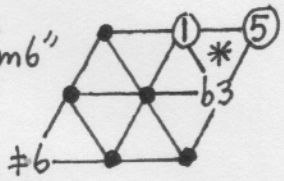
Septimal Dom7□
Harmonic Series Tetrad
"German Sixth Chord"

1	3	5	#6 –
b6	1	b3	#4 – "G."
4	6	1	#2 –
♯2	b5	♯6	1 –



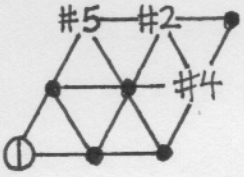
Septimal m6□
Subharmonic Series Tetrad

1	b3	5	♯6 – "m6"
6	1	3	b5 –
4	b6	1	♯2 –
#2	#4	#6	1 –



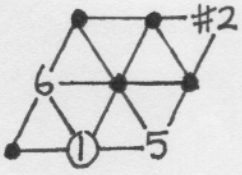
Dissonant Dom7□

1	♯3	5	b7 –
#5	1	#2	#4 –
4	♯6	1	b3 –
2	b5	6	1 –



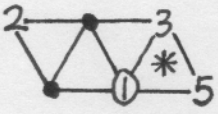
Dissonant m6□

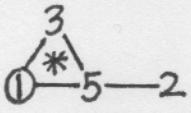
1	#2	5	6 –
♯6	1	♯3	b5 –
4	#5	1	2 –
b3	#4	b7	1 –



Major Triad "add nine" (M9□)

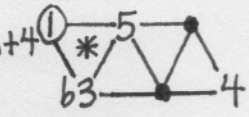
1	2	3	5 – I+2
b7	1	2	4 –
b6	b7	1	b3 –
4	5	6	1 –

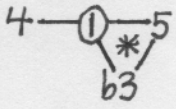




Minor Triad "add Four" (eleven) (m11□)

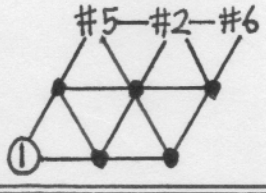
1	b3	4	5 – m+4
6	1	2	3 –
5	b7	1	2 –
4	b6	b7	1 –





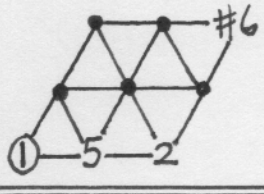
Septimal Major Δ "add nine" □

1	2	♯3	5 –
b7	1	♯2	4 –
#5	#6	1	#2 –
4	5	♯6	1 –



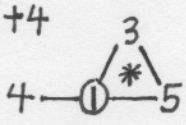
Septimal Minor Δ "add Four" □

1	#2	4	5 –
♯6	1	♯2	♯3 –
5	#6	1	2 –
4	#5	b7	1 –



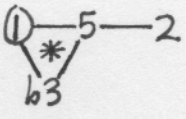
Major Δ "add Four" (M11□)

1	3	4	5 – I+4
b6	1	b2	b3 –
5	7	1	2 –
4	6	b7	1 –



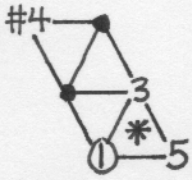
Minor Triad "add nine" (m9□)

1	2	b3	5 – m+2
b7	1	b2	4 –
6	7	1	3 –
4	5	b6	1 –



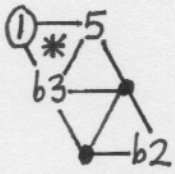
Major Five-Flat-Five □ (M5b5□)

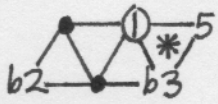
1	3	#4	5 –
b6	1	2	b3 –
b5	b7	1	b2 –
4	6	7	1 –



Minor Five-Flat-Nine □ (m5b9□)

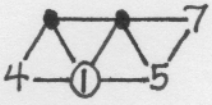
1	b2	b3	5 –
7	1	2	#4 –
6	b7	1	3 –
4	b5	b6	1 –





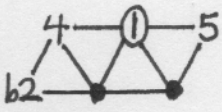
Major-Seven, Suspended 4 □ (M7s4□)

1	4	5	7 –
5	1	2	#4 –
4	b7	1	3 –
b2	b5	b6	1 –

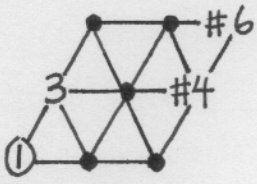


Flat-two, suspended 4 □ (b2s4□)

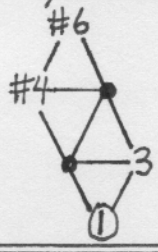
1	b2	4	5 –
7	1	3	#4 –
5	b6	1	2 –
4	b5	b7	1 –



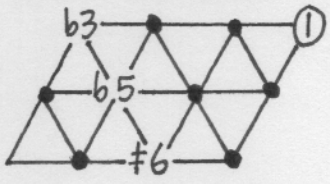
Seven-Flat-Five □ (7b5□) "French Sixth Chord" – Symmetrical



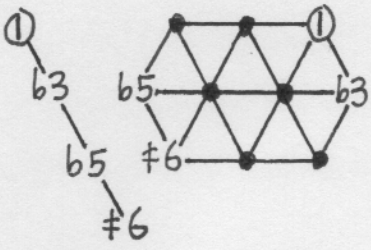
1	2	#4	b6 –
b7	1	3	b5 – I7b5□
b5	b6	1	#2 –
3	#4	#6	1 –

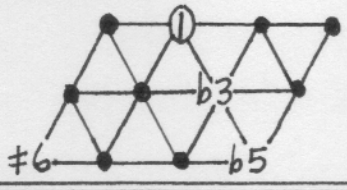


Diminished □ (°□) – Symmetrical



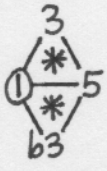
1	b3	b5	#6 –
6	1	b3	b5 –
#4	6	1	b3 –
#2	#4	6	1 –





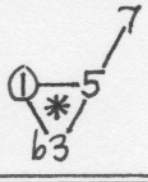
Major-Minor Tetrad (Mm□) – Symmetrical

1	b3	3	5 –
6	1	#1	3 –
b6	b7	1	b3 –
4	b6	6	1 –



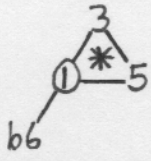
Minor Natural-Seven □ (mb7□)

1	b3	5	7 –
6	1	3	#5 –
4	b6	1	3 –
b2	b4	b6	1 –



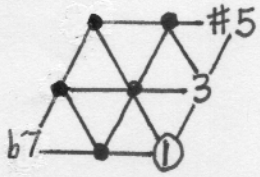
Major Flat-six □ (Mb6□)

1	3	5	b6 –
b6	1	b3	b4 –
4	6	1	b2 –
3	#5	7	1 –



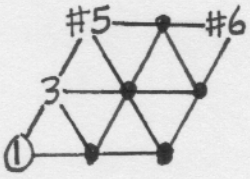
Seven-Sharp-Five □ ① (7#5□)

1	3	#5	b7 –
b6	1	3	b5 –
b4	b6	1	♯2 –
2	#4	#6	1 –



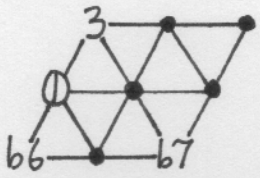
7#5 □ ②

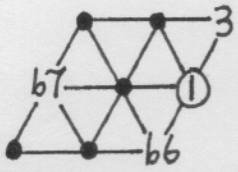
1	3	#5	#6 –
b6	1	3	#4 –
b4	b6	1	2 –
♯2	b5	b7	1 –



7#5 □ ③

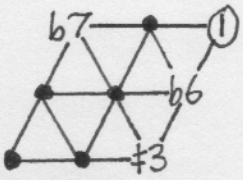
1	3	b6	b7 –
b6	1	b4	b5 –
3	#5	1	2 –
2	#4	b7	1 –

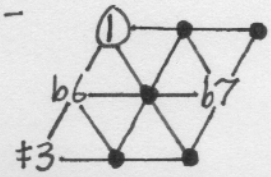




7#5 □ ④

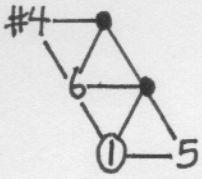
1	♯3	b6	b7 –
#5	1	3	#4 –
3	b6	1	2 –
2	b5	b7	1 –

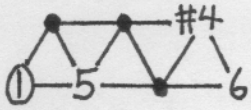




Six, Five, Flat-Five □ (6, 5b5□)

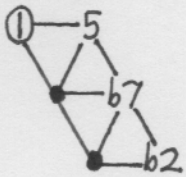
1	#4	5	6 –
b5	1	b2	b3 –
4	7	1	2 –
b3	6	b7	1 –

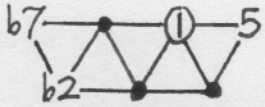


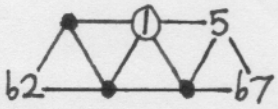


Seven, Five, Flat-nine □ (7, 5b9□)

1	b2	5	b7 –
7	1	#4	6 –
4	b5	1	b3 –
2	b3	6	1 –

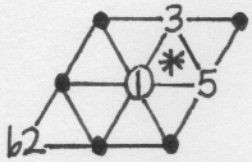






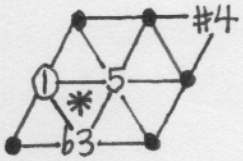
Major Flat-nine □ (Mb9□)

1	b2	3	5 –
7	1	#2	#4 –
b6	♯6	1	b3 –
4	b5	6	1 –



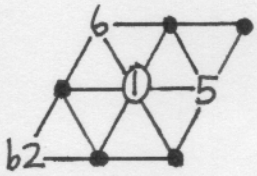
Minor Five-Flat-Five □ (m5b5□)

1	b3	#4	5 –
6	1	#2	3 –
b5	♯6	1	b2 –
4	b6	7	1 –



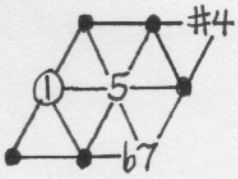
Six, Five-Flat-nine □ (6,5b9□)

1	b2	5	6 –
7	1	#4	#5 –
4	b5	1	2 –
b3	b4	b7	1 –



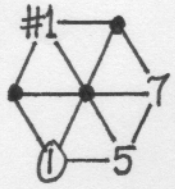
Seven, Five-Flat-Five □ (m5b5□)

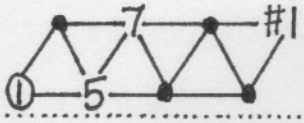
1	#4	5	b7 –
b5	1	b2	b4 –
4	7	1	b3 –
2	#5	6	1 –



Five, Major-Seven, Flat-nine □ ①
(5, M7, b9 □)

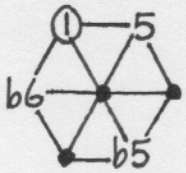
1	#1	5	7 –
b1	1	b5	b7 –
4	#4	1	3 –
b2	2	b6	1 –

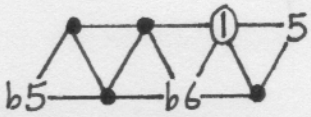




Flat-six, Five-Flat-Five □ ①
(b6, 5b5 □)

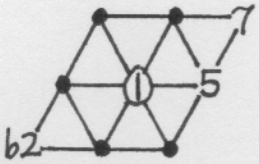
1	b5	5	b6 –
#4	1	#1	2 –
4	b1	1	b2 –
3	b7	7	1 –





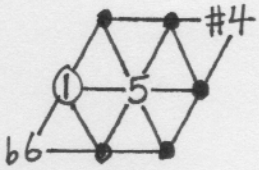
5, M7, b9 □ ②

1	b2	5	7 –
7	1	#4	#6 –
4	b5	1	3 –
b2	♯2	b6	1 –



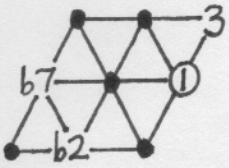
b6, 5b5 □ ②

1	#4	5	b6 –
b5	1	b2	bb3 –
4	7	1	b2 –
3	#6	7	1 –



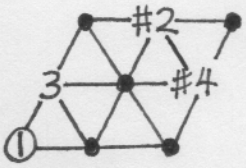
Dominant Seven "add Flat-nine" □
(Dom 7, b9 □)
Diminished Scalar Segment

1	b2	3	b7 –
7	1	#2	6 –
b6	bb7	1	b5 –
2	b3	#4	1 –



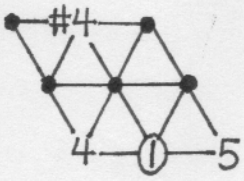
Major-minor Three, Flat-Five □
(Mm 3, b5 □)
Diminished Scalar Segment

1	#2	3	#4 –
♯6	1	b2	b3 –
b6	7	1	2 –
b5	6	b7	1 –



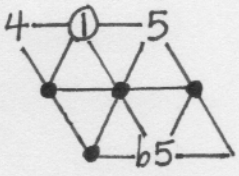
Five, Flat-Five, Suspended Four □ ①
(5, b5s4 □)

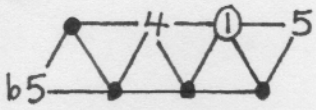
1	4	#4	5 –
5	1	#1	2 –
b5	b1	1	b2 –
4	b7	7	1 –



Five, Flat-Five, Suspended Four □ ②
(5, b5s4 □)

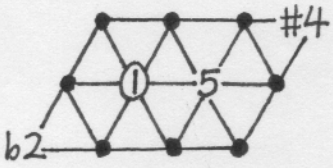
1	4	b5	5 –
5	1	b2	2 –
#4	7	1	#1 –
4	b7	b1	1 –





Five, Flat-Five, Flat-nine Tetrad (5, b5, b9 □) – Symmetrical

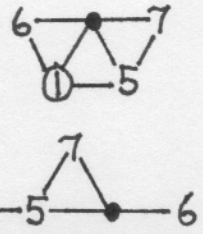
1	b2	#4	5 –
7	1	#3	#4 –
b5	♯5	1	b2 –
4	b5	7	1 –



Scalar Tetrads

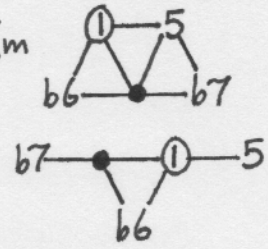
Major Diatonic Tetrad (M dia.□)

1	5	6	7 – IM
4	1	2	3 –
b3	b7	1	2 –
b2	b6	b7	1 –



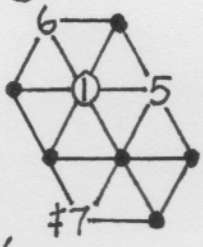
Minor Diatonic □ (m dia.□)

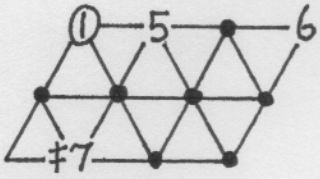
1	5	b6	b7 – Im
4	1	b2	b3 –
3	7	1	2 –
2	6	b7	1 –



Septimal M dia. □ ①

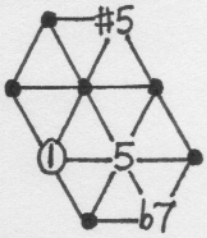
1	5	6	♯7 –
4	1	2	♯3 –
b3	b7	1	♯2 –
♯1	♯5	♯6	1 –

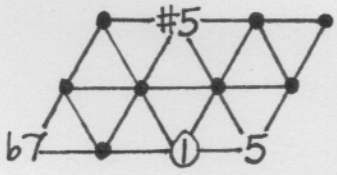




Septimal Minor dia. □ ①

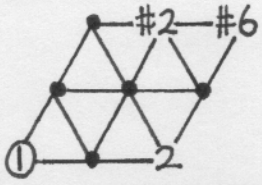
1	5	♯5	b7 –
4	1	♯1	b3 –
b4	b1	1	♯2 –
2	6	♯6	1 –





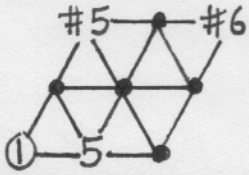
Septimal M dia. □ ②

1	5	♯6	♯7 –
4	1	♯2	♯3 –
♯2	♯6	1	2 –
♯1	♯5	b7	1 –

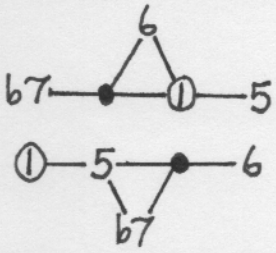


Septimal Minor dia. □ ②

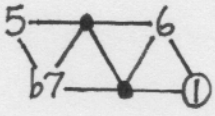
1	5	♯5	♯6 –
4	1	♯1	♯2 –
b4	b1	1	2 –
♯2	♯6	b7	1 –

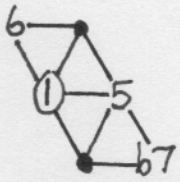


Five, Six, Seven Scalar □ (5,6,7□) – Symmetrical



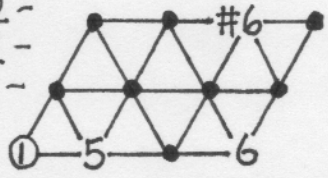
1	5	6	b7 –
4	1	2	b3 –
b3	b7	1	b2 –
2	6	7	1 –





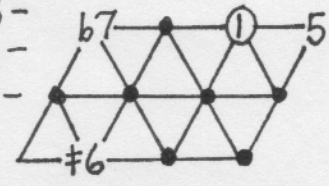
Septimal 5,6,7□ ①

1	5	6	♯6 –
4	1	2	♯2 –
b3	b7	1	♯1 –
♯2	♯6	b1	1 –



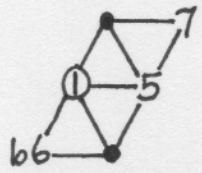
Septimal 5,6,7□ ②

1	5	♯6	b7 –
4	1	♯2	b3 –
♯2	♯6	1	♯1 –
2	6	♯7	1 –



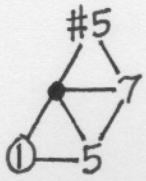
Major-Seven, Flat-six Scalar Tetrad (M7,b6□) – Symmetrical

1	5	b6	7–
4	1	b2	3–
3	7	1	#2–
b2	b6	♯6	1–



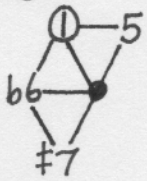
Septimal M7,b6□ ①

1	5	#5	7–
4	1	#1	3–
b4	b1	1	b3–
b2	b6	6	1–

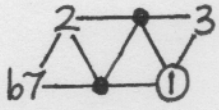


Septimal M7,b6□ ②

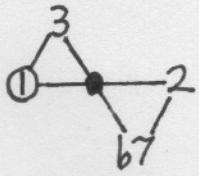
1	5	b6	♯7–
4	1	b2	♯3–
3	7	1	b3–
#1	#5	6	1–



Whole-tone Scalar Tetrad (W.T.□) – Symmetrical

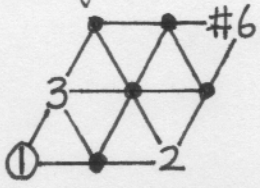


1	2	3	#4–
b7	1	2	3 – I7,9□
b6	b7	1	2–
b5	b6	b7	1–



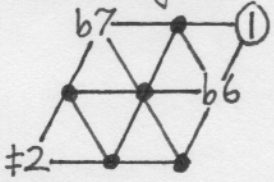
Septimal W.T.□ ①
(Harmonic Series segment)

1	2	3	#6–
b7	1	2	#5–
b6	b7	1	#4–
♯2	b4	b5	1–



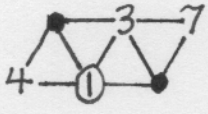
Septimal W.T.□ ②
(Sub-harmonic Series segment)

1	♯2	b6	b7–
#6	1	#4	#5–
3	b5	1	2–
2	b4	b7	1–



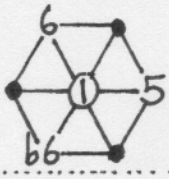
Major-seven, Five-Flat-Five Scalar□ (M7,5b5□) – Symmetrical

1	3	4	7–
b6	1	b2	5–
5	7	1	#4–
b2	4	b5	1–



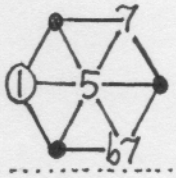
Five, Six, Flat-six Scalar□ (5,6,b6□)
(Classical Greek Chromatic Tetrachord)

1	5	b6	6–
4	1	b2	2–
3	7	1	#1–
b3	b7	b1	1–



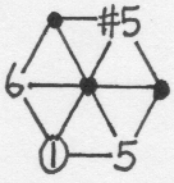
Five, Major-minor Seven□ (5,Mm7□)

1	5	b7	7–
4	1	b3	3–
2	6	1	#1–
b2	b6	b1	1–



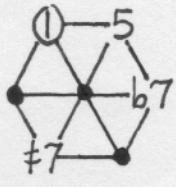
Septimal 5,6,b6□

1	5	#5	6–
4	1	#1	2–
b4	b1	1	b2–
b3	b7	7	1–



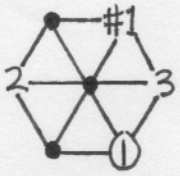
Septimal 5,Mm7□

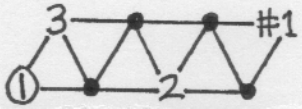
1	5	b7	♯7–
4	1	b3	♯3–
2	6	1	b2–
#1	#5	7	1–



Flat-two, Two, Three Cluster □
(b2, 2, 3 □)

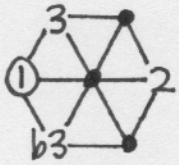
1	#1	2	3 –
b1	1	b2	b3 –
b7	7	1	2 –
b6	6	b7	1 –

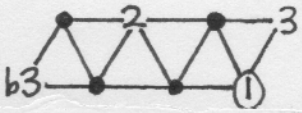




Two, Major-minor Three Cluster □
(2, Mm3 □)

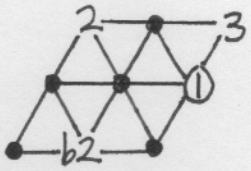
1	2	b3	3 –
b7	1	b2	2 –
6	7	1	#1 –
b6	b7	b1	1 –

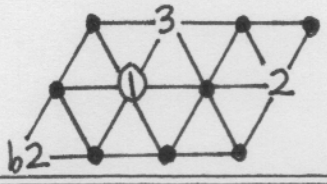




Septimal b2, 2, 3 □

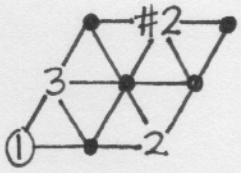
1	b2	2	3 –
7	1	#1	#2 –
b7	b1	1	2 –
b6	♯6	b7	1 –

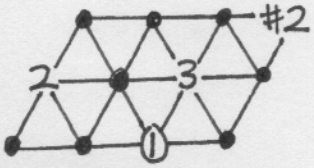




Septimal 2, Mm3 □

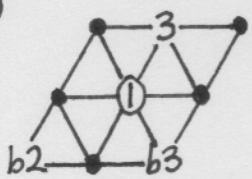
1	2	#2	3 –
b7	1	#1	2 –
♯6	b1	1	b2 –
b6	b7	7	1 –





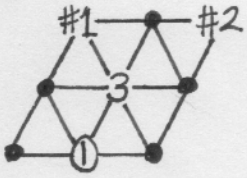
Flat-two, Major-Minor Three Cluster
(b2, Mm3 □) ①

1	b2	b3	3 –
7	1	2	#2 –
6	b7	1	#1 –
b6	♯6	b1	1 –



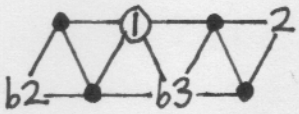
Flat-two, Major-minor Three Cluster
(b2, Mm3 □) ②

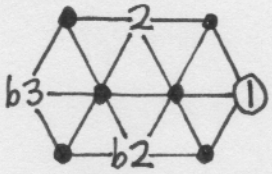
1	#1	#2	3 –
b1	1	2	b3 –
♯6	b7	1	b2 –
b6	6	7	1 –



Chromatic Tetrad (Chrom □) – Symmetrical

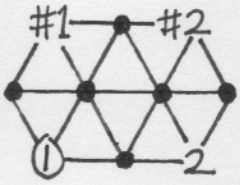
1	b2	2	b3 –
7	1	#1	2 –
b7	b1	1	b2 –
6	b7	7	1 –





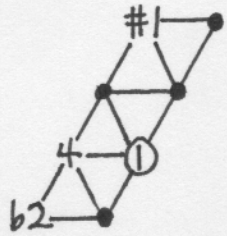
Septimal Chromatic Tetrad – Symmetrical □

1	#1	2	#2 –
b1	1	b2	2 –
b7	7	1	#1 –
♯6	b7	b1	1 –



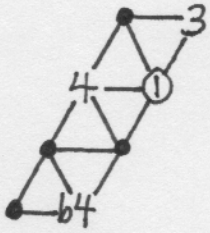
Archytas Enharmonic □
(Arch. Enh. □)

1	#1	b2	4 -
b1	1	♯1	♯3 -
7	♮1	1	3 -
5	#5	b6	1 -



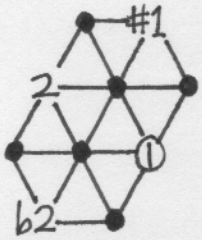
Inverse Archytas Enharmonic □
(Inv. Arch. □)

1	3	b4	4 -
b6	1	♯1	b2 -
#5	♮1	1	#1 -
5	7	b1	1 -



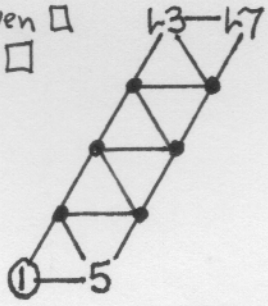
Enharmonic Tetrad (Enh. □) - Symmetrical

1	#1	b2	2 -
b1	1	♯1	b2 -
7	♮1	1	#1 -
b7	7	b1	1 -



A very 'distant' □ - The Neutral Tetrad (Symmetrical)

1	♮3	5	♮7	- Neutral Seven □
♮6	1	♮3	5	- Neutral Six □
4	♮6	1	♮3	-
♮2	4	♮6	1	-



The Pentads ☆

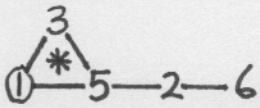
Major-seven, nine ☆ (Ptolemaic Pentad ①)

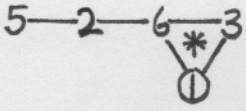
1	2	3	5	7	– IM7,9
b7	1	2	4	6	–
b6	b7	1	b3	5	– Im7,b6
4	5	6	1	3	– IM6,4
b2	b3	4	b6	1	–

Minor-seven, nine ☆ (Ptolemaic Pentad ②)

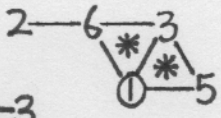
1	2	b3	5	b7	– Im7,9
b7	1	b2	4	b6	–
6	7	1	3	5	– IM6,M7
4	5	b6	1	b3	– Imb6,4
2	3	4	6	1	–

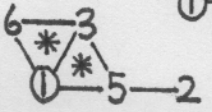
Major-six, nine ☆ (Pentatonic Scales) – Symmetrical

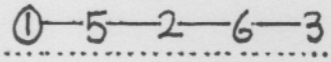




1	2	3	5	6 –
b7	1	2	4	5 –
b6	b7	1	b3	4 –
4	5	6	1	2 –
b3	4	5	b7	1 –

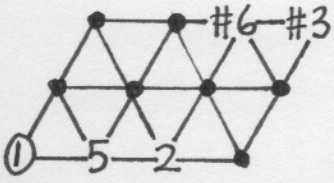






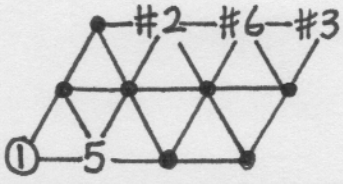
Septimal Pentatonic ☆ ①

1	♯2	♯3	5	♯6 –
#6	1	2	#3	5 –
#5	b7	1	#2	4 –
4	♯5	♯6	1	♯2 –
#2	4	5	#6	1 –



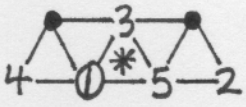
Septimal Pentatonic ☆ ②

1	2	♯3	5	♯6 –
b7	1	♯2	4	♯5 –
#5	#6	1	#2	4 –
4	5	♯6	1	♯2 –
#2	#3	5	#6	1 –



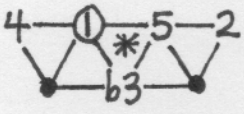
Major Scalar ☆ (Msc.☆)

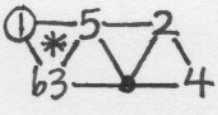
1	2	3	4	5 –
b7	1	2	b3	4 –
b6	b7	1	b2	b3 –
5	6	7	1	2 –
4	5	6	b7	1 –



Minor Scalar ☆ (msc.☆)

1	2	b3	4	5 –
b7	1	b2	b3	4 –
6	7	1	2	3 –
5	6	b7	1	2 –
4	5	b6	b7	1 –



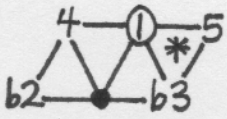


Lydian Scalar ☆

1	2	3	#4	5 -
b7	1	2	3	4 -
b6	b7	1	2	b3 -
b5	b6	b7	1	b2 -
4	5	6	7	1 -

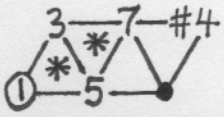
Phrygian Scalar ☆

1	b2	b3	4	5 -
7	1	2	3	#4 -
6	b7	1	2	3 -
5	b6	b7	1	2 -
4	b5	b6	b7	1 -



Olympus Greek scale ☆
Chinese + Koto Scale

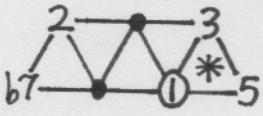
1	2	b3	5	b6 - Imb6,9
b7	1	b2	4	b5 -
6	7	1	3	4 -
4	5	b6	1	b2 -
3	#4	5	7	1 - IM7,#4

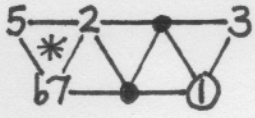


Major-seven, eleven ☆
"Balinese" scale

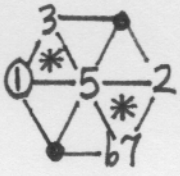
1	3	4	5	7 -
b6	1	b2	b3	5 - Imb6,b9
5	7	1	2	#4 -
4	6	b7	1	3 -
b2	4	b5	b6	1 -

Dominant Seven, nine ☆ — Symmetrical
Minor-sixth, eleven ☆



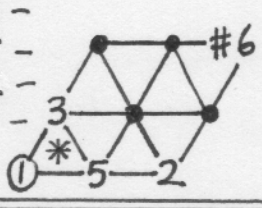


1	2	3	5	b7 - I^{9}_{7}
b7	1	2	4	b6 -
b6	b7	1	b3	b5 -
4	5	6	1	b3 - Im6,11
2	3	#4	6	1 -



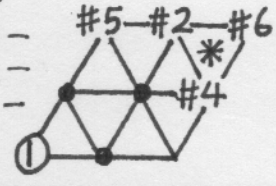
Harmonic Series ☆

1	2	3	5	#6 - $I^{\#6}$
b7	1	2	4	#5 -
b6	b7	1	b3	#4 -
4	5	6	1	#2 -
♯2	♯3	b5	♯6	1 -

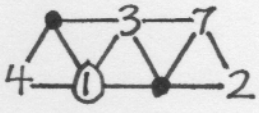


Sub-harmonic Series ☆

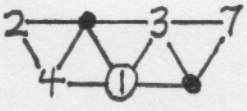
1	b3	4	5	♯6 - Im
6	1	2	3	b5 -
5	b7	1	2	♯3 -
4	b6	b7	1	♯2 -
#2	#4	#5	#6	1 -



Dominant Scalar Pentad — Symmetrical ☆

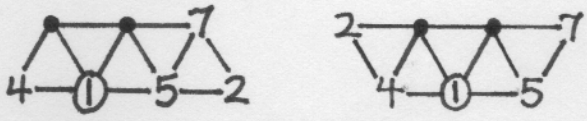


1	2	3	4	7 -
b7	1	2	b3	6 -
b6	b7	1	b2	5 -
5	6	7	1	#4 -
b2	b3	4	b5	1 -



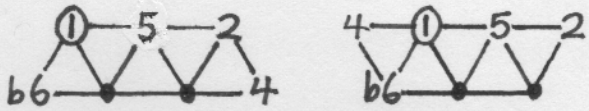
Suspended □ "add M7" ☆

1	2	4	5	7 –
b7	1	b3	4	6 –
5	6	1	2	#4 –
4	5	b7	1	3 –
b5	b3	b5	b6	1 –



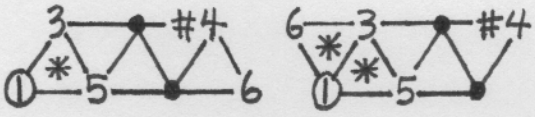
Suspended □ "add m6" ☆

1	2	4	5	b6 –
b7	1	b3	4	b5 –
5	6	1	2	b3 –
4	5	b7	1	b2 –
3	#4	6	7	1 –



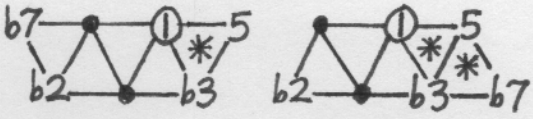
Major-sixth □ "add #4"
Minor-sixth □ "add b7" ☆

1	3	#4	5	6 – IM6, #4
b6	1	2	b3	4 –
b5	b7	1	b2	b3 –
4	6	7	1	2 –
b3	5	6	b7	1 – Im6, b7



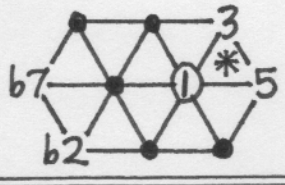
Minor-seventh □ "add b2"
Major-sixth □ "add b7" ☆

1	b2	b3	5	b7 – Im7, b9
7	1	2	#4	6 –
6	b7	1	3	5 – I7, M6
4	b5	b6	1	b3 –
2	b3	4	6	1 –



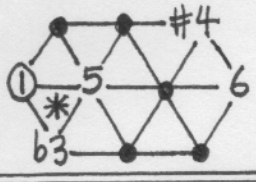
Dominant-seven □ "add b9"
Diminished □ superset ☆

1	b2	3	5	b7 – I7, b9
7	1	#2	#4	6 –
b6	#6	1	b3	b5 –
4	b5	6	1	b3 –
2	b3	#4	6	1 –



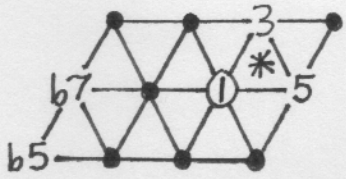
Minor-sixth □ "add #4"
Diminished □ superset ☆

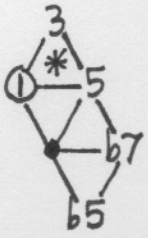
1	b3	#4	5	6 – Im6, #4
6	1	#2	3	#4 –
b5	#6	1	b2	b3 –
4	b6	7	1	2 –
b3	b5	6	b7	1 –



Dominant-seven □ "add b5" ☆
7b5 □ superset

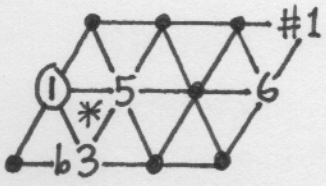
1	3	b5	5	b7	– I^7b5
b6	1	♯2	b3	b5	–
#4	#6	1	#1	3	–
4	6	b1	1	b3	–
2	#4	b6	6	1	–

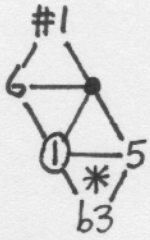




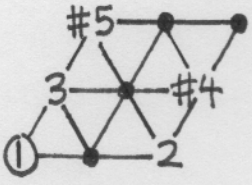
Minor-sixth □ "add b2" ☆
7b5 □ superset

1	#1	b3	5	6	– Im6, b9
b1	1	♯2	b5	b6	–
6	#6	1	3	#4	–
4	#4	b6	1	2	–
b3	3	b5	b7	1	–

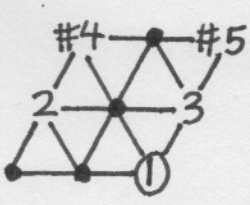




Wholetone Scalar ☆ ① – Symmetrical

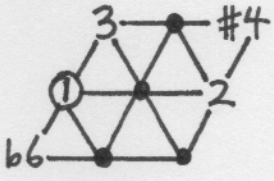


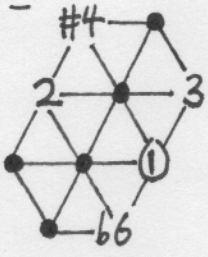
1	2	3	#4	#5	–
b7	1	2	3	#4	–
b6	b7	1	2	3	–
b5	b6	b7	1	2	–
b4	b5	b6	b7	1	–



Wholetone Scalar ☆ ②

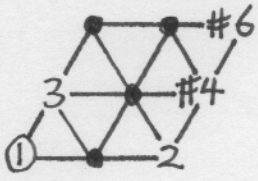
1	2	3	#4	b6	–
b7	1	2	3	b5	–
b6	b7	1	2	b4	–
b5	b6	b7	1	♯2	–
3	#4	#5	#6	1	–

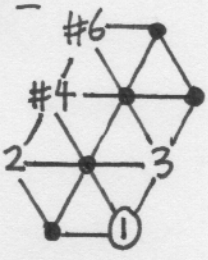




Wholetone Scalar ☆ ③

1	♯2	♯3	b5	b6	–
#6	1	2	3	#4	–
#5	b7	1	2	3	–
#4	b6	b7	1	2	–
3	b5	b6	b7	1	–

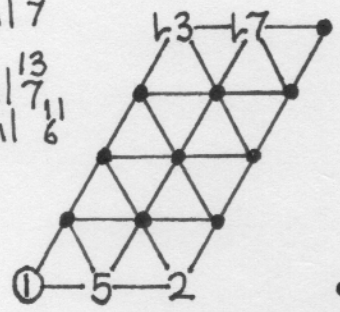


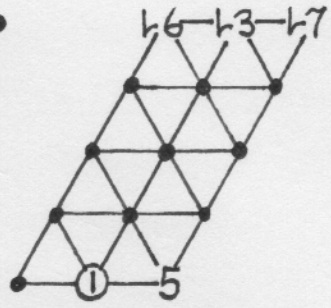


A very 'distant' altered Pentad - The Neutral ☆

1	2	♭3	5	♭7	- Neutral 9/7
♭7	1	♭2	4	♭6	-
♭6	♭7	1	♭3	5	- Neutral 13/7
4	5	♭6	1	♭3	- Neutral 11/6
♭2	♭3	4	♭6	1	-

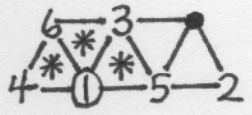
Symmetrical

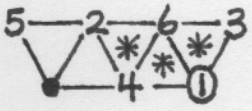




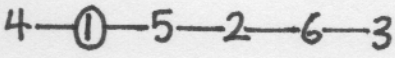
The Hexads

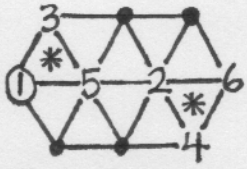
The Hexachord (Ptolemaic Hexad #1) - Symmetrical





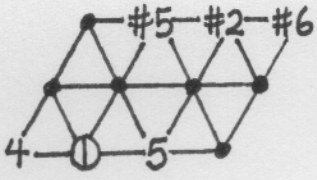
1	2	3	4	5	6	- IM6, 9, 11
b7	1	2	b3	4	5	- Im7, 9, 11
b6	b7	1	b2	b3	4	-
5	6	7	1	2	3	- IM6, M7, 9
4	5	6	b7	1	2	-
b3	4	5	b6	b7	1	- Im7, 11, b13





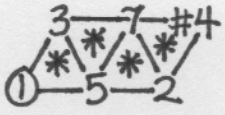
The Septimal Hexachord - Symmetrical

1	♯2	♯3	4	5	♯6 -
#6	1	2	#2	#3	5 -
#5	b7	1	#1	#2	4 -
5	♯6	♯7	1	2	♯3 -
4	♯5	♯6	b7	1	♯2 -
#2	4	5	#5	#6	1 -



Ptolemaic Hexad #2 - Symmetrical

1	3	4	5	6	7	- IM6, M7, 11
b6	1	b2	b3	4	5	- Imb6, b9, 11
5	7	1	2	3	#4	- IM7, 9, #11
4	6	b7	1	2	3	-
b3	5	b6	b7	1	2	- Im7, b6, 9
b2	4	b5	b6	b7	1	-

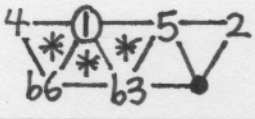


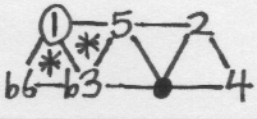
Ptolemaic Heptad Subset

1	2	3	4	5	7	- IM7, 9, 11
b7	1	2	b3	4	6	-
b6	b7	1	b2	b3	5	- Im7, b9, b13
5	6	7	1	2	#4	-
4	5	6	b7	1	3	- I7, 11, 13
b2	b3	4	b5	b6	1	-

Ptolemaic Heptad Subset

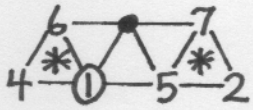
1	2	b3	4	5	b6	- Imb6, 9, 11
b7	1	b2	b3	4	b5	-
6	7	1	2	3	4	-
5	6	b7	1	2	b3	- Im7, 9, 13
4	5	b6	b7	1	b2	-
3	#4	5	6	7	1	- IM6, M7, #11





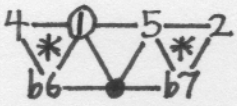
Ptolemaic Heptad Subset

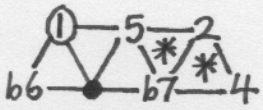
1	2	4	5	6	7 -
b7	1	b3	4	5	6 - Im7,6,11
5	6	1	2	3	#4 - IM6,9,#11
4	5	b7	1	2	3 - I7,9,11
b3	4	b6	b7	1	2 -
b2	b3	b5	b6	b7	1 -



Ptolemaic Heptad Subset

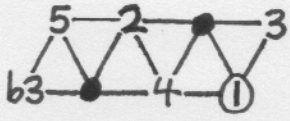
1	2	4	5	b6	b7 -
b7	1	b3	4	b5	b6 -
5	6	1	2	b3	4 - Im6,9,11
4	5	b7	1	b2	b3 - Im7,b9,11
3	#4	6	7	1	2 -
2	3	5	6	b7	1 - I7,6,9

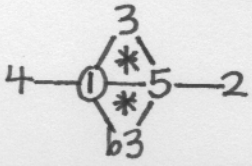




Symmetrical Scalar Hexad

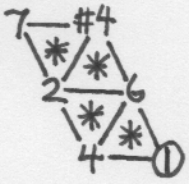
1	2	b3	3	4	5 -
b7	1	b2	2	b3	4 -
6	7	1	#1	2	3 -
b6	b7	b1	1	b2	b3 -
5	6	b7	7	1	2 -
4	5	b6	6	b7	1 -





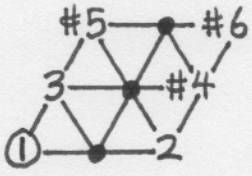
Symmetrical Hexad

1	2	4	#4	6	7 -
b7	1	b3	3	5	6 -
5	6	1	#1	3	#4 - IM6,b9,#11
b5	b6	b1	1	b3	4 -
b3	4	b6	6	1	2 -
b2	b3	b5	5	b7	1 - Im7,b9,#11



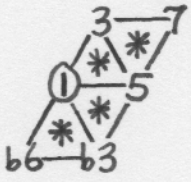
Wholetone Scalar Hexad - Symmetrical

1	2	3	#4	#5	#6 -
b7	1	2	3	#4	#5 -
b6	b7	1	2	3	#4 -
b5	b6	b7	1	2	3 -
b4	b5	b6	b7	1	2 -
#2	b4	b5	b6	b7	1 -



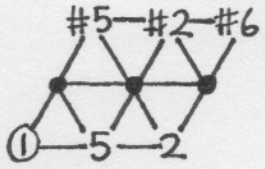
Symmetrical Septimal Hexad

1	b3	3	5	b6	7 -
6	1	#1	3	4	#5 -
b6	b1	1	b3	b4	5 -
4	b6	6	1	b2	3 -
3	5	#5	7	1	#2 -
b2	#3	4	b6	#6	1 -



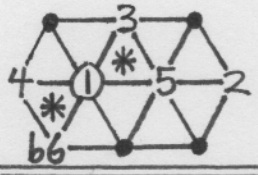
Symmetrical Septimal Hexad

1	#1	#2	4	5	#5	–
b1	1	2	#3	b5	5	–
#6	b7	1	#2	#3	4	–
5	#5	#6	1	2	#2	–
4	#4	#5	b7	1	#1	–
#3	4	5	#6	#7	1	–



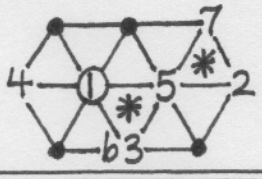
Scalar Hexad

1	2	3	4	5	b6	– IMb6,9,11
b7	1	2	b3	4	b5	–
b6	b7	1	b2	b3	b4	–
5	6	7	1	2	b3	– Im6,47,9
4	5	6	b7	1	b2	–
3	#4	#5	6	7	1	–



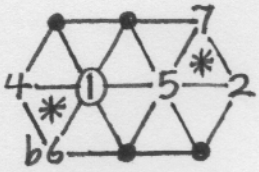
Scalar Hexad

1	2	b3	4	5	7	– Im47,9,11
b7	1	b2	b3	4	6	–
6	7	1	2	3	#5	–
5	6	b7	1	2	#4	–
4	5	b6	b7	1	3	– I^7,11,b13
b2	b3	b4	b5	b6	1	–



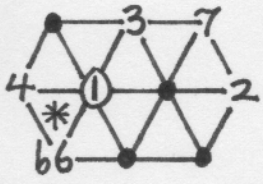
Symmetrical Diminished □ Superset

1	2	4	5	b6	7	–
b7	1	b3	4	b5	6	–
5	6	1	2	b3	#4	– Im6,9,#11
4	5	b7	1	b2	3	– I^7,b9,11
3	#4	6	7	1	#2	–
b2	b3	b5	b6	bb7	1	–



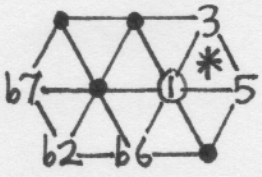
Diminished □ Superset ①

1	2	3	4	b6	7	–
b7	1	2	b3	b5	6	–
b6	b7	1	b2	b4	5	–
5	6	7	1	b3	#4	– Im6,47,#11
3	#4	#5	6	1	#2	–
b2	b3	4	b5	#6	1	–



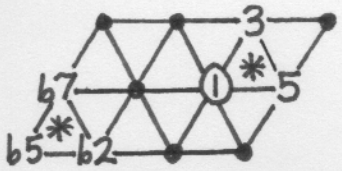
Diminished □ Superset ②

1	b2	3	5	b6	b7	– I^7,b9,b13
7	1	#2	#4	5	6	–
b6	bb7	1	b3	b4	b5	–
4	b5	6	1	b2	b3	–
3	4	#5	7	1	2	–
2	b3	#4	6	b7	1	–



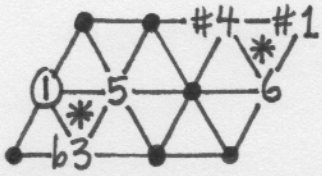
Dominant seven, b9, #11

1	b2	3	b5	5	b7	– I^7, b9, #11
7	1	#2	4	#4	6	–
b6	♯6	1	♯2	b3	b5	–
#4	5	#6	1	#1	3	– I^7, b9, #11
4	b5	6	♯7	1	b3	–
2	b3	#4	b6	6	1	–



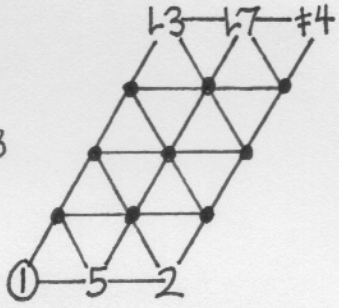
Minor six, b9, #11

1	#1	b3	#4	5	6	– Im6, b9, #11
♯7	1	♯2	4	b5	b6	–
6	#6	1	#2	3	#4	–
b5	5	♯6	1	b2	b3	– Im6, b9, #11
4	#4	b6	7	1	2	–
b3	3	b5	6	b7	1	–



A very 'distant' Hexad – The Neutral ✡ Symmetrical

1	2	↓3	♯4	5	↓7	– Neutral 7, 9, 11
b7	1	↓2	↓3	4	↓6	–
↓6	↓7	1	2	↓3	5	– Neutral 7, 9, 13
↓5	↓6	b7	1	↓2	4	–
4	5	↓6	↓7	1	↓3	– Neutral 7, 11, 13
↓2	↓3	4	5	↓6	1	– Neutral 6, 9, 11

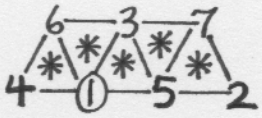


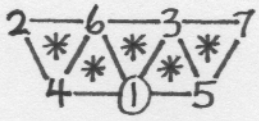
The Heptads

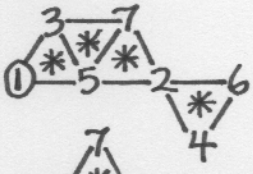
The relatively large number of tabulated heptads reflects the common usage of these patterns as scale types.

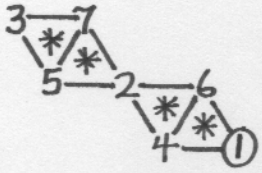
The Diatonic Scales (Ptolemaic Heptads) - Symmetrical

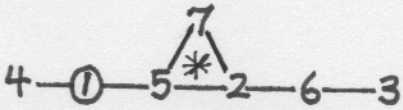
1	2	3	4	5	6	7	- Ionian
b7	1	2	b3	4	5	6	- Dorian
b6	b7	1	b2	b3	4	5	- Phrygian
5	6	7	1	2	3	#4	- Lydian
4	5	6	b7	1	2	3	- Mixolydian
b3	4	5	b6	b7	1	2	- Aeolian
b2	b3	4	b5	b6	b7	1	- Locrian

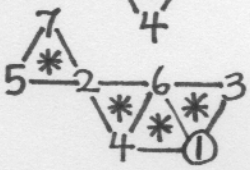


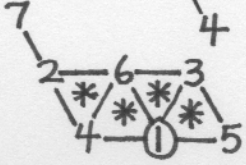


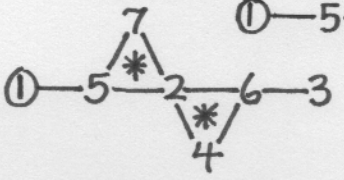


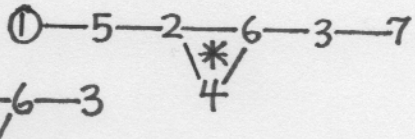


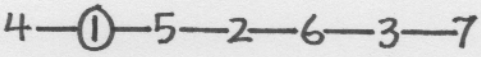






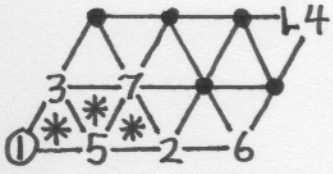






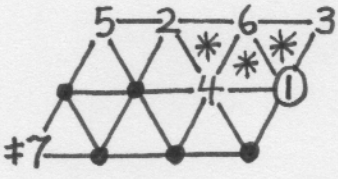
Septimal Diatonic Scale ①

1	2	3	↓4	5	6	7	-
b7	1	2	↓3	4	5	6	-
b6	b7	1	↓2	b3	4	5	-
♯5	♯6	♯7	1	♯2	♯3	♯♯4	-
4	5	6	#6	1	2	3	-
b3	4	5	↓6	b7	1	2	-
b2	b3	4	↓5	b6	b7	1	-



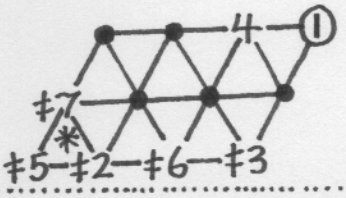
Septimal Diatonic Scale ②

1	2	3	4	5	6	♯7	-
b7	1	2	b3	4	5	♯6	-
b6	b7	1	b2	b3	4	♯5	-
5	6	7	1	2	3	♯♯4	-
4	5	6	b7	1	2	♯3	-
b3	4	5	b6	b7	1	♯2	-
↓2	↓3	↓4	↓5	↓6	↓7	1	-



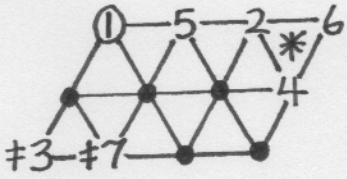
Septimal Diatonic Scale ③

1	‡2	‡3	4	‡5	‡6	‡7 –
‡7	1	2	‡3	4	5	6 –
‡6	‡7	1	‡2	‡3	4	5 –
5	‡6	‡7	1	‡2	‡3	#4 –
‡4	5	6	‡7	1	2	3 –
‡3	4	5	‡6	‡7	1	2 –
‡2	‡3	4	‡5	‡6	‡7	1 –



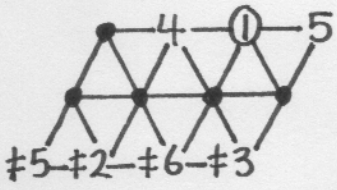
Septimal Diatonic Scale ④

1	2	‡3	4	5	6	‡7 –
‡7	1	‡2	‡3	4	5	‡6 –
‡6	‡7	1	‡2	‡3	‡4	5 –
5	6	‡7	1	2	3	#4 –
4	5	‡6	‡7	1	2	‡3 –
‡3	4	‡5	‡6	‡7	1	‡2 –
‡2	‡3	4	‡5	‡6	‡7	1 –



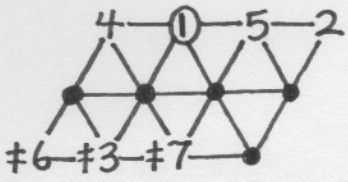
Septimal Diatonic Scale ⑤

1	‡2	‡3	4	5	‡6	‡7 –
‡7	1	2	‡3	‡4	5	6 –
‡6	‡7	1	‡2	‡3	4	5 –
5	‡6	‡7	1	2	‡3	#4 –
4	‡5	‡6	‡7	1	‡2	‡3 –
‡3	4	5	‡6	‡7	1	2 –
‡2	‡3	4	‡5	‡6	‡7	1 –



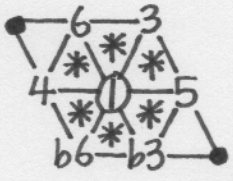
Septimal Diatonic Scale ⑥

1	2	‡3	4	5	‡6	‡7 –
‡7	1	‡2	‡3	4	‡5	‡6 –
‡6	‡7	1	‡2	‡3	‡4	5 –
5	6	‡7	1	2	‡3	#4 –
4	5	‡6	‡7	1	‡2	‡3 –
‡3	‡4	5	‡6	‡7	1	2 –
‡2	‡3	4	‡5	‡6	‡7	1 –



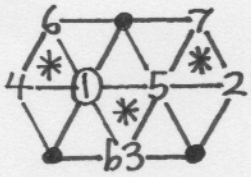
Ellis' "Harmonic Heptad" – Symmetrical Core Chromatic pattern

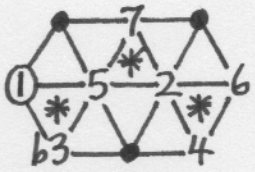
1	b3	3	4	5	b6	6 –
6	1	#1	2	3	4	#4 –
b6	b1	1	b2	b3	b4	4 –
5	b7	7	1	2	b3	3 –
4	b6	6	b7	1	b2	2 –
3	5	#5	6	7	1	‡2 –
b3	b5	5	b6	b7	b1	1 –



Ascending Minor Scale – Symmetrical Major-Minor Scale

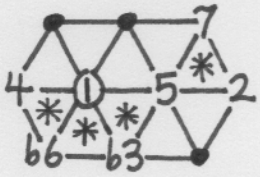
1	2	3	4	5	b6	b7	– Mm. sc.
b7	1	2	b3	4	b5	b6	–
b6	b7	1	b2	b3	b4	b5	–
5	6	7	1	2	b3	4	– Asc. m. sc.
4	5	6	b7	1	b2	b3	–
3	#4	#5	6	7	1	2	–
2	3	#4	5	6	b7	1	– Combined Lydian-Mixolydian





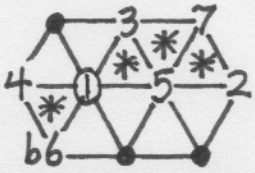
Harmonic Minor Scale

1	2	b3	4	5	b6	7	– H.m.
b7	1	b2	b3	4	b5	6	–
6	7	1	2	3	4	#5	–
5	6	b7	1	2	b3	#4	–
4	5	b6	b7	1	b2	3	–
3	#4	5	6	7	1	#2	–
b2	b3	b4	b5	b6	bb7	1	–



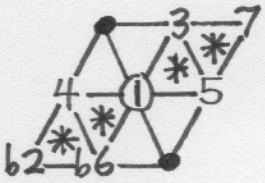
Major Flat-six Scale

1	2	3	4	5	b6	7	– Mb6 sc.
b7	1	2	b3	4	b5	6	–
b6	b7	1	b2	b3	b4	5	–
5	6	7	1	2	b3	#4	–
4	5	6	b7	1	b2	3	–
3	#4	#5	6	7	1	#2	–
b2	b3	4	b5	b6	♯6	1	–



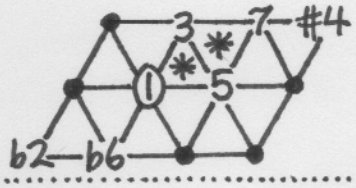
Bhairav-type scale – symmetrical Hungarian Gypsy scale

1	b2	3	4	5	b6	7	– Bhairav
7	1	#2	3	#4	5	#6	–
b6	bb7	1	b2	b3	b4	5	–
5	b6	7	1	2	b3	#4	– Balkans
4	b5	6	b7	1	b2	3	–
3	4	#5	6	7	1	#2	–
b2	bb3	4	b5	b6	bb7	1	–



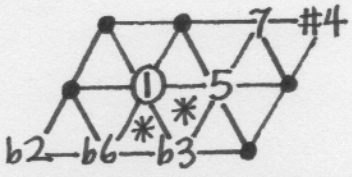
Purvi-type scale ①

1	b2	3	#4	5	b6	7 – Pu.
7	1	#2	#3	#4	5	#6 –
b6	bb7	1	2	b3	b4	5 –
b5	bb6	b7	1	b2	bb3	4 –
4	b5	6	b1	1	b2	3 –
3	4	#5	#6	7	1	#2 –
b2	bb3	4	5	b6	bb7	1 –



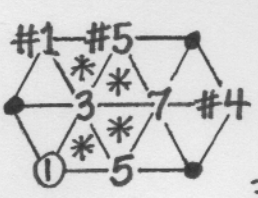
Todi-type scale ①

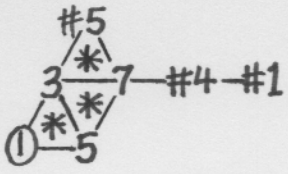
1	b2	b3	#4	5	b6	7 – To.
7	1	2	#3	#4	5	#6 –
6	b7	1	#2	3	4	#5 –
b5	bb6	bb7	1	b2	bb3	4 –
4	b5	b6	7	1	b2	3 –
3	4	5	#6	7	1	#2 –
b2	bb3	b4	5	b6	bb7	1 –

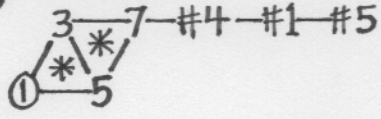


Purvi-type scale ②
Greek Didymus Chromatic

1	#1	3	#4	5	#5	7 – Pu.
b1	1	b3	4	b5	5	b7 –
b6	6	1	2	b3	3	5 –
b5	5	b7	1	b2	2	4 –
4	#4	6	7	1	#1	3 –
b4	4	b6	b7	b1	1	b3 –
b2	2	4	5	b6	6	1 –

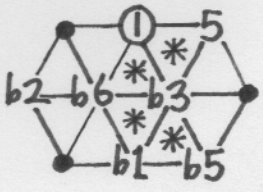


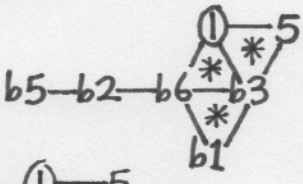


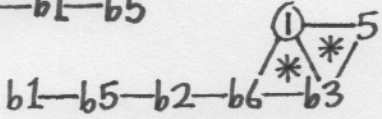


Todi-type scale ②
Inverse Didymus Chromatic

1	b2	b3	b5	5	b6	b1 – To.
7	1	2	4	#4	5	b7 –
6	b7	1	b3	3	4	b6 –
#4	5	6	1	#1	2	4 –
4	b5	b6	b1	1	b2	b4 –
3	4	5	b7	7	1	b3 –
#1	2	3	5	#5	6	1 –

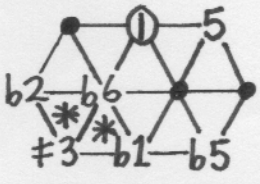


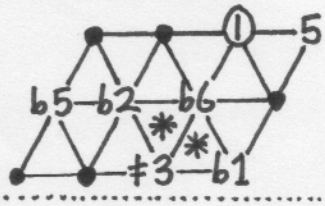




Purvi-type scale ③
Ptolemy Soft Chromatic

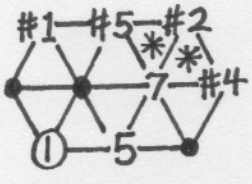
1	b2	♯3	b5	5	b6	b1 – Pu.
7	1	b3	4	#4	5	b7 –
#5	6	1	2	#2	3	5 –
#4	5	b7	1	#1	2	4 –
4	b5	bb7	b1	1	b2	b4 –
3	4	b6	b7	7	1	b3 –
#1	2	4	5	#5	6	1 –

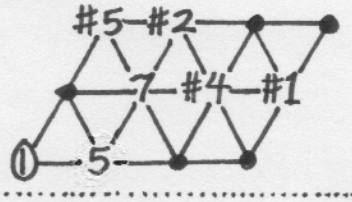




Todi-type scale ③
Inverse Ptolemy Soft Chromatic

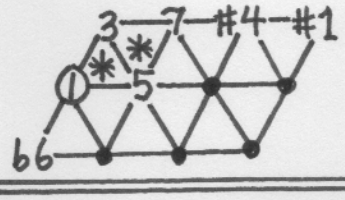
1	#1	#2	#4	5	#5	7 – To.
b1	1	2	4	b5	5	b7 –
bb7	b7	1	b3	b4	4	b6 –
b5	5	6	1	b2	2	4 –
4	#4	#5	7	1	#1	3 –
b4	4	5	b7	b1	1	b3 –
b2	2	3	5	b6	6	1 –





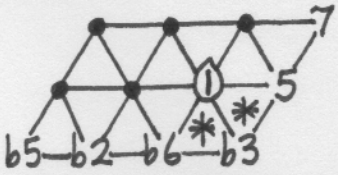
Purvi-type scale ④

1	#1	3	#4	5	b6	7 – Pu.
b1	1	b3	4	b5	bb6	b7 –
b6	6	1	2	b3	b4	5 –
b5	5	b7	1	b2	bb3	4 –
4	#4	6	7	1	b2	3 –
3	#3	#5	#6	7	1	#2 –
b2	2	4	5	b6	bb7	1 –



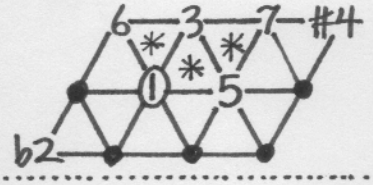
Todi-type scale ④

1	b2	b3	b5	5	b6	7 – To.
7	1	2	4	#4	5	#6 –
6	b7	1	b3	3	4	#5 –
#4	5	6	1	#1	2	#3 –
4	b5	b6	b1	1	b2	3 –
3	4	5	b7	7	1	#2 –
b2	bb3	b4	bb6	b6	bb7	1 –



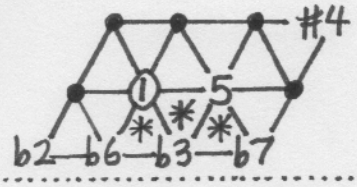
Marva-type scale ①

1	b2	3	#4	5	6	7 – Ma.
7	1	#2	#3	#4	#5	#6 –
b6	bb7	1	2	b3	4	5 –
b5	bb6	b7	1	b2	b3	4 –
4	b5	6	7	1	2	3 –
b3	b4	5	6	b7	1	2 –
b2	bb3	4	5	b6	b7	1 –



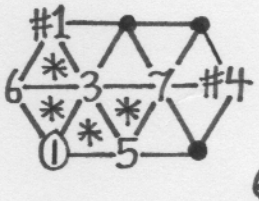
Oriental-type scale ①

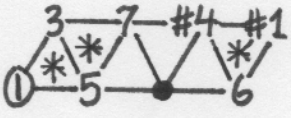
1	b2	b3	#4	5	b6	b7 – Or.
7	1	2	#3	#4	5	6 –
6	b7	1	#2	3	4	5 –
b5	bb6	bb7	1	b2	bb3	b4 –
4	b5	b6	7	1	b2	b3 –
3	4	5	#6	7	1	2 –
2	b3	4	#5	6	b7	1 –

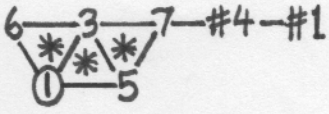


Marva-type scale ②

1	#1	3	#4	5	6	7 – Ma.
b1	1	b3	4	b5	b6	b7 –
b6	6	1	2	b3	4	5 –
b5	5	b7	1	b2	b3	4 –
4	#4	6	7	1	2	3 –
b3	3	5	6	b7	1	2 –
b2	2	4	5	b6	b7	1 –

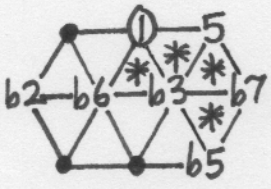


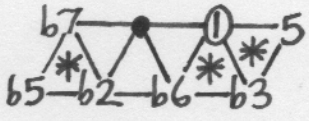


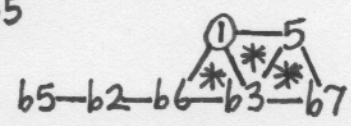


Oriental-type scale ②

1	b2	b3	b5	5	b6	b7 – Or.
7	1	2	4	#4	5	6 –
6	b7	1	b3	3	4	5 –
#4	5	6	1	#1	2	3 –
4	b5	b6	b1	1	b2	b3 –
3	4	5	b7	7	1	2 –
2	b3	4	b6	6	b7	1 –

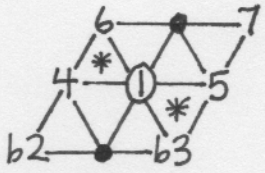






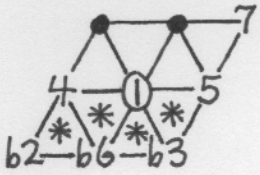
The Augmented Heptad - Symmetrical

1	b2	b3	4	5	6	7 –
7	1	2	3	#4	#5	#6 –
6	b7	1	2	3	#4	#5 –
5	b6	b7	1	2	3	#4 –
4	b5	b6	b7	1	2	3 –
b3	b4	b5	b6	b7	1	2 –
b2	bb3	b4	b5	b6	b7	1 –



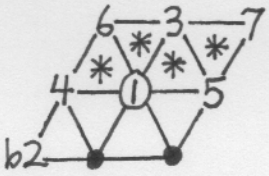
Neapolitan Scale ①

1	b2	b3	4	5	b6	7 –
7	1	2	3	#4	5	#6 –
6	b7	1	2	3	4	#5 –
5	b6	b7	1	2	b3	#4 –
4	b5	b6	b7	1	b2	3 –
3	4	5	6	7	1	#2 –
b2	bb3	b4	b5	b6	bb7	1 –



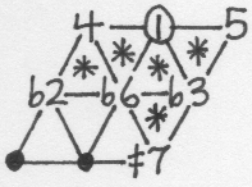
Neapolitan (Inverse) Scale ①

1	b2	3	4	5	6	7 –
7	1	#2	3	#4	#5	#6 –
b6	bb7	1	b2	b3	4	5 –
5	b6	7	1	2	3	#4 –
4	b5	6	b7	1	2	3 –
b3	b4	5	b6	b7	1	2 –
b2	bb3	4	b5	b6	b7	1 –



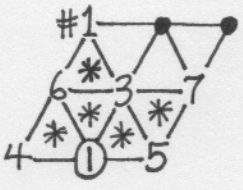
Neapolitan Scale ②

1	b2	b3	4	5	b6	♯7 –
7	1	2	3	#4	5	b7 –
6	b7	1	2	3	4	b6 –
5	b6	b7	1	2	b3	b5 –
4	b5	b6	b7	1	b2	b4 –
3	4	5	6	7	1	b3 –
#1	2	3	#4	#5	6	1 –



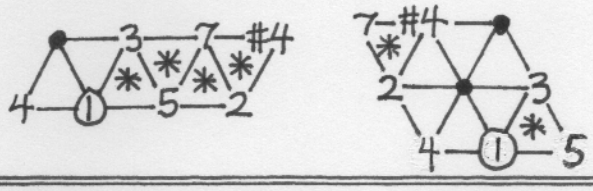
Inverse Neapolitan Scale ②

1	#1	3	4	5	6	7 –
b1	1	b3	b4	b5	b6	b7 –
b6	6	1	b2	b3	4	5 –
5	#5	7	1	2	3	#4 –
4	#4	6	b7	1	2	3 –
b3	3	5	b6	b7	1	2 –
b2	2	4	b5	b6	b7	1 –



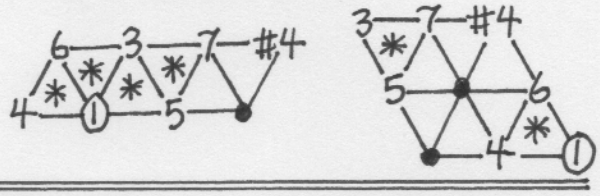
Ptolemaic Ogdoad Subset

1	2	3	4	#4	5	7 –
b7	1	2	b3	3	4	6 –
b6	b7	1	b2	2	b3	5 –
5	6	7	1	#1	2	#4 –
b5	b6	b7	b1	1	b2	4 –
4	5	6	b7	7	1	3 –
b2	b3	4	b5	5	b6	1 –



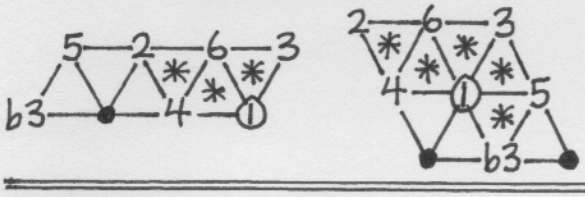
Ptolemaic Ogdoad Subset

1	3	4	#4	5	6	7 –
b6	1	b2	2	b3	4	5 –
5	7	1	#1	2	3	#4 –
b5	b7	b1	1	b2	b3	4 –
4	6	b7	7	1	2	3 –
b3	5	b6	6	b7	1	2 –
b2	4	b5	5	b6	b7	1 –



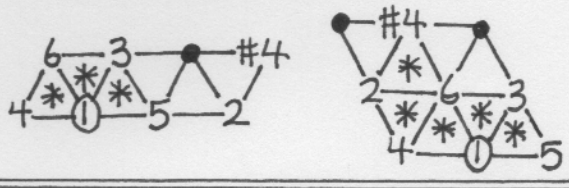
Ptolemaic Ogdoad Subset

1	2	b3	3	4	5	6 –
b7	1	b2	2	b3	4	5 –
6	7	1	#1	2	3	#4 –
b6	b7	b1	1	b2	b3	4 –
5	6	b7	7	1	2	3 –
4	5	b6	6	b7	1	2 –
b3	4	b5	5	b6	b7	1 –



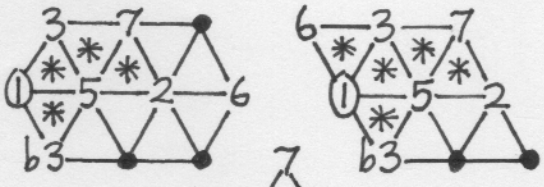
Ptolemaic Ogdoad Subset

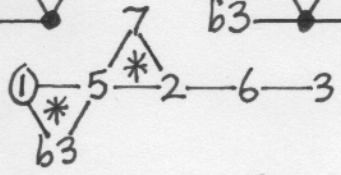
1	2	3	4	#4	5	6 –
b7	1	2	b3	3	4	5 –
b6	b7	1	b2	2	b3	4 –
5	6	7	1	#1	2	3 –
b5	b6	b7	b1	1	b2	b3 –
4	5	6	b7	7	1	2 –
b3	4	5	b6	6	b7	1 –

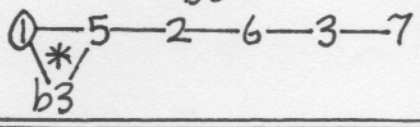


Chromatic-genus Heptad

1	2	b3	3	5	6	7 –
b7	1	b2	2	4	5	6 –
6	7	1	#1	3	#4	#5 –
b6	b7	b1	1	b3	4	5 –
4	5	b6	6	1	2	3 –
b3	4	b5	5	b7	1	2 –
b2	b3	b4	4	b6	b7	1 –

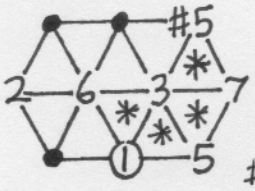


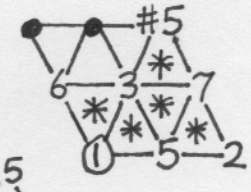


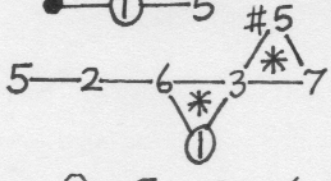


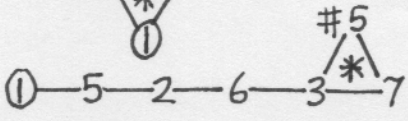
Chromatic-genus Heptad

1	2	3	5	#5	6	7 –
b7	1	2	4	#4	5	6 –
b6	b7	1	b3	3	4	5 –
4	5	6	1	#1	2	3 –
b4	b5	b6	b1	1	b2	b3 –
b3	4	5	b7	7	1	2 –
b2	b3	4	b6	6	b7	1 –



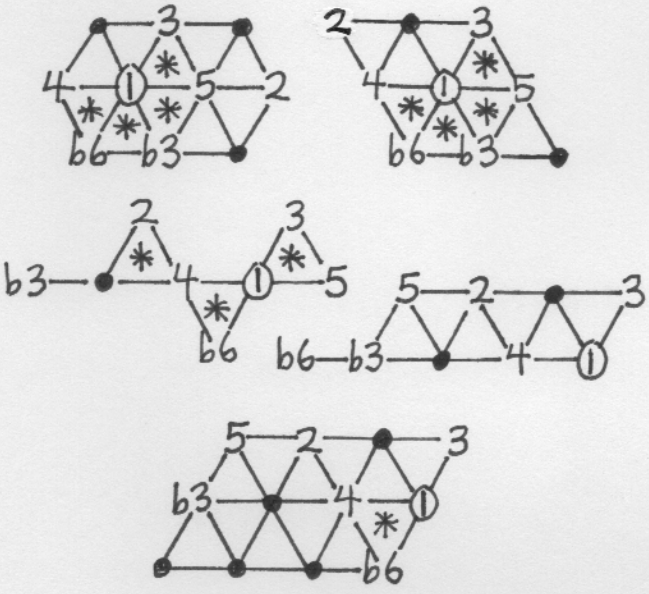






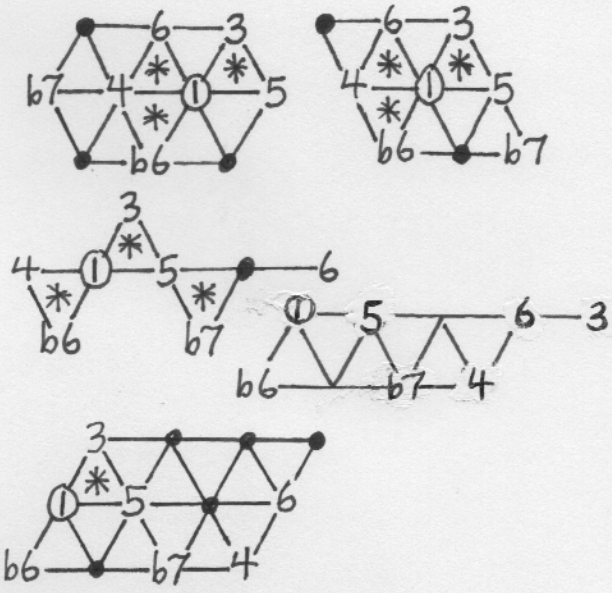
Chromatic-genus Heptad

1	2	b3	3	4	5	b6 -
b7	1	b2	2	b3	4	b5 -
6	7	1	#1	2	3	4 -
b6	b7	b1	1	b2	b3	b4 -
5	6	b7	7	1	2	b3 -
4	5	b6	6	b7	1	b2 -
3	#4	5	#5	6	7	1 -



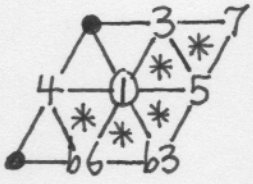
Chromatic-genus Heptad

1	3	4	5	b6	6	b7 -
b6	1	b2	b3	b4	4	b5 -
5	7	1	2	b3	3	4 -
4	6	b7	1	b2	2	b3 -
3	#5	6	7	1	#1	2 -
b3	5	b6	b7	b1	1	b2 -
2	#4	5	6	b7	7	1 -



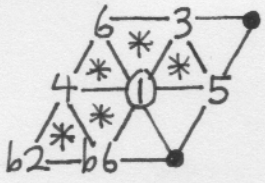
Chromatic-genus Heptad

1	b3	3	4	5	b6	7 -
6	1	#1	2	3	4	#5 -
b6	b1	1	b2	b3	b4	5 -
5	b7	7	1	2	b3	#4 -
4	b6	6	b7	1	b2	3 -
3	5	#5	6	7	1	#2 -
b2	b4	4	b5	b6	bb7	1 -



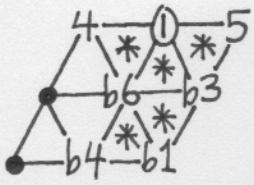
Chromatic-genus Heptad

1	b2	3	4	5	b6	6 -
7	1	#2	3	#4	5	#5 -
b6	bb7	1	b2	b3	b4	4 -
5	b6	7	1	2	b3	3 -
4	b5	6	b7	1	b2	2 -
3	4	#5	6	7	1	#1 -
b3	b4	5	b6	b7	b1	1 -



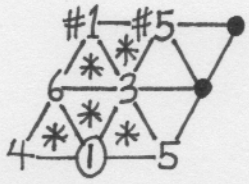
Chromatic-genus Heptad

1	b3	b4	4	5	b6	b1 –
6	1	b2	2	3	4	b6 –
#5	7	1	#1	#2	3	5 –
5	b7	b1	1	2	b3	b5 –
4	b6	bb7	b7	1	b2	b4 –
3	5	b6	6	7	1	b3 –
#1	3	4	#4	#5	6	1 –



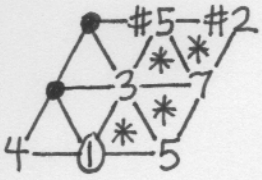
Chromatic-genus Heptad

1	#1	3	4	5	#5	6 –
b1	1	b3	b4	b5	5	b6 –
b6	6	1	b2	b3	3	4 –
5	#5	7	1	2	#2	3 –
4	#4	6	b7	1	#1	2 –
b4	4	b6	bb7	b1	1	b2 –
b3	3	5	b6	b7	7	1 –



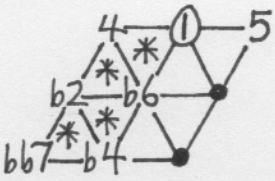
Chromatic-genus Heptad

1	#2	3	4	5	#5	7 –
bb7	1	b2	bb3	b4	4	b6 –
b6	7	1	b2	b3	3	5 –
5	#6	7	1	2	#2	#4 –
4	#5	6	b7	1	#1	3 –
b4	5	b6	bb7	b1	1	b3 –
b2	3	4	b5	b6	6	1 –



Chromatic-genus Heptad

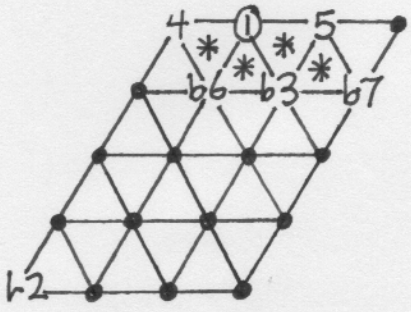
1	b2	b4	4	5	b6	bb7 –
7	1	b3	3	#4	5	b6 –
#5	6	1	#1	#2	3	4 –
5	b6	b1	1	2	b3	b4 –
4	b5	bb7	b7	1	b2	bb3 –
3	4	b6	6	7	1	b2 –
#2	3	5	#5	#6	7	1 –



Some supplimentary 'distant' Heptads with qualities loosely described as 'Arabic' i.e. using '3/4th tones'. All are symmetrical.

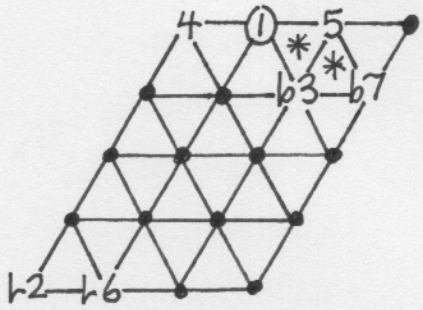
Arabic Scale #1:

1	r2	b3	4	5	b6	b7 –	Minor
r7	1	r2	r3	#4	r5	r6 –	
6	r7	1	2	3	4	5 –	Major
5	r6	b7	1	2	b3	4 –	Minor
4	r5	b6	b7	1	b2	b3 –	
3	#4	5	6	7	1	2 –	Major
2	r3	4	5	6	b7	1 –	Neutral



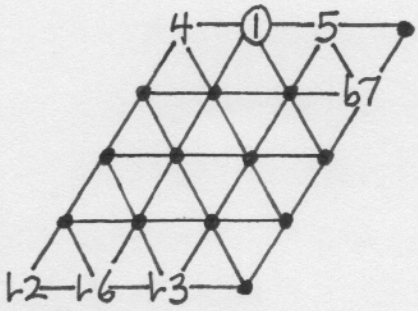
Arabic Scale #2: Ptolemy's Diatonic Hemiolon (symmetrical)

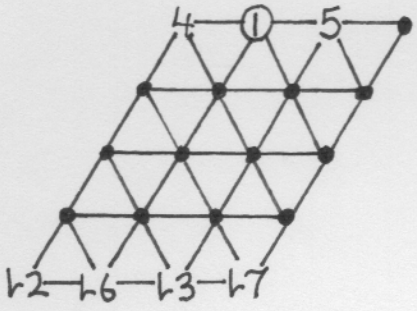
1	r2	b3	4	5	r6	b7	- Minor
r7	1	r2	r3	♯4	5	r6	- Neutral
6	r7	1	2	3	♯4	5	- Major
5	r6	b7	1	2	r3	4	- Neutral
4	r5	b6	b7	1	r2	b3	-
r3	4	r5	r6	r7	1	r2	-
2	r3	4	5	6	r7	1	- Neutral



Arabic Scale #3: A very 'distant' symmetrical harmony

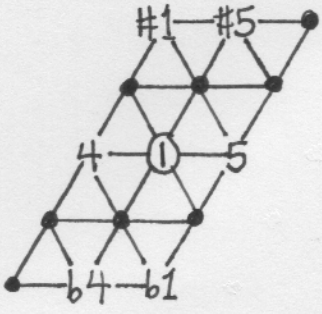
1	r2	r3	4	5	r6	b7	- Neutral
r7	1	2	r3	♯4	5	r6	- Neutral
r6	b7	1	r2	r3	4	r5	-
5	r6	r7	1	2	r3	4	- Neutral
4	r5	r6	b7	1	r2	b3	-
r3	4	5	r6	r7	1	r2	- Neutral
2	r3	♯4	5	6	r7	1	- Neutral





An altered form of **Bhairav**: -Symmetrical

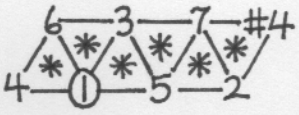
1	#1	b4	4	5	#5	b1	- "major"
b1	1	r3	b4	b5	5	r7	- Neutral
#5	r6	1	#1	#2	r3	5	- Neutral
5	#5	b1	1	2	#2	b5	- "minor"
4	#4	♯6	b7	1	#1	b4	-
b4	4	r6	♯6	b1	1	r3	-
#1	r2	4	#4	#5	r6	1	-

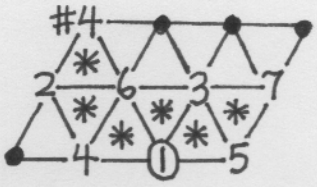


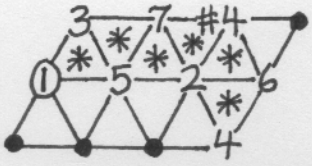
A few prominant Ogdoads

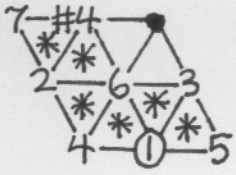
The (full) Diatonic Ogdoad – Symmetrical Ptolemaic Ogdoad

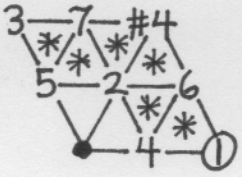
1	2	3	4	#4	5	6	7	– Lyd. + Ion.
b7	1	2	b3	3	4	5	6	– Mix. + Dor.
b6	b7	1	b2	2	b3	4	5	– Aeo. + Phr.
5	6	7	1	#1	2	3	#4	– * + Lyd.
b5	b6	b7	b1	1	b2	b3	4	– Loc. + *
4	5	6	b7	7	1	2	3	– Ion. + Mix.
b3	4	5	b6	6	b7	1	2	– Dor. + Aeo.
b2	b3	4	b5	5	b6	b7	1	– Phr. + Loc.

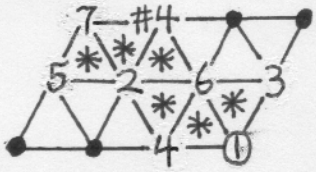






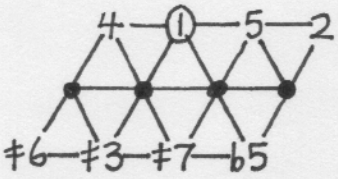






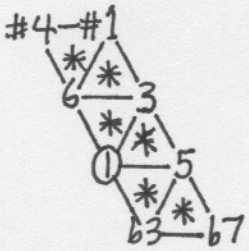
The Septimal Diatonic Ogdoad – Symmetrical

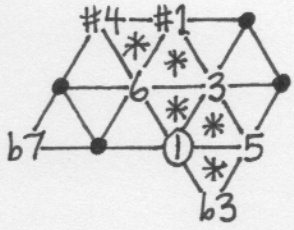
1	2	♯3	4	b5	5	♯6	♯7	–
b7	1	♯2	b3	b4	4	♯5	♯6	–
#5	#6	1	#1	2	#2	4	5	–
5	6	b1	1	b2	2	♯3	#4	–
#4	#5	b7	7	1	#1	b3	4	–
4	5	♯6	b7	b1	1	♯2	♯3	–
#2	#3	5	#5	6	#6	1	2	–
#1	#2	4	#4	5	#5	b7	1	–

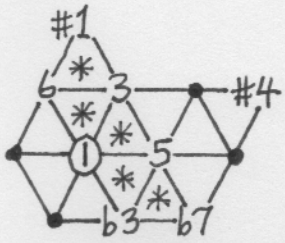


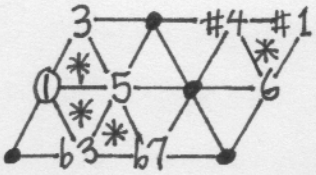
Diminished Scale ① – Symmetrical

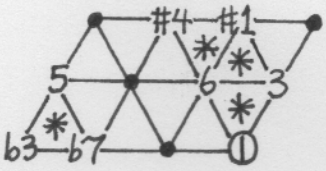
1	#1	b3	3	#4	5	6	b7	–
b1	1	#2	b3	4	b5	b6	#6	–
6	#6	1	#1	#2	3	#4	5	–
b6	6	b1	1	2	b3	4	b5	–
b5	5	#6	b7	1	b2	b3	b4	–
4	#4	b6	6	7	1	2	b3	–
b3	3	b5	5	6	b7	1	b2	–
2	#2	4	#4	#5	6	7	1	–





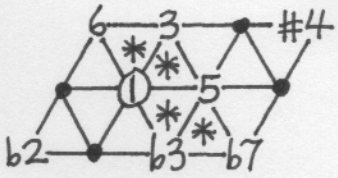


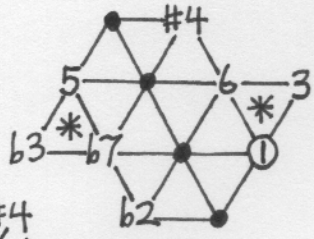


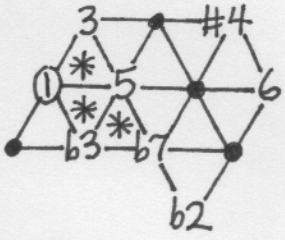


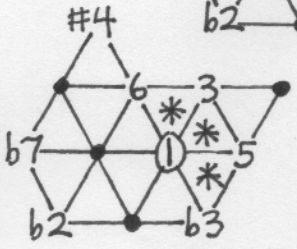
Diminished Scale ② – Symmetrical

1	b2	b3	3	#4	5	6	b7	–
7	1	2	#2	#3	#4	#5	6	–
6	b7	1	#1	#2	3	#4	5	–
b6	bb7	b1	1	2	b3	4	b5	–
b5	bb6	bb7	b7	1	b2	b3	b4	–
4	b5	b6	6	7	1	2	b3	–
b3	b4	b5	5	6	b7	1	b2	–
2	b3	4	#4	#5	6	7	1	–



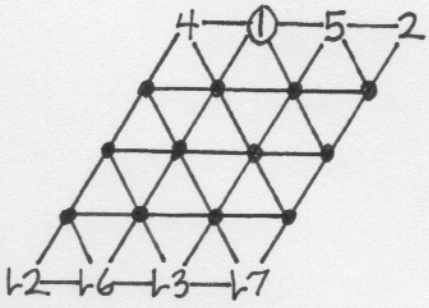






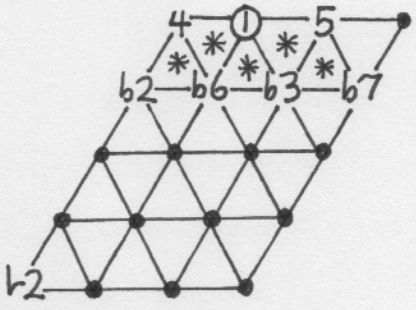
A very 'distant' altered-diatonic Ogdoad
The Neutral Diatonic Ogdoad – Symmetrical

1	♭2	2	♭3	4	5	♭6	♭7	– Neutral
♭7	1	♯1	2	♭3	♯4	5	6	– Neutral
b7	b1	1	b2	b3	4	b5	b6	–
b6	b7	b7	1	b2	b3	4	5	– Neutral
5	b6	6	b7	1	2	b3	♯4	– Neutral
4	b5	5	b6	b7	1	b2	b3	– Neutral
b3	4	♯4	5	b6	b7	1	2	– Neutral
b2	b3	b3	4	b5	b6	b7	1	–

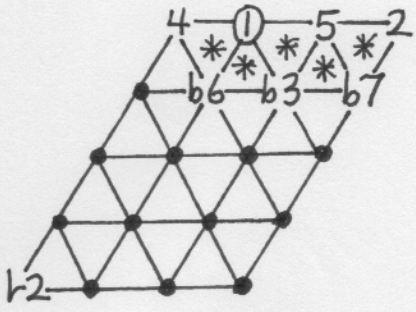


Next, we have a small group (3 pairs) of ogdoads which are non-symmetrical. They are derived from the Heptads: 'Arabic' Scale #1, #2, and #3. The first pair, shown below, can also be derived from the standard Diatonic Hep'tad.

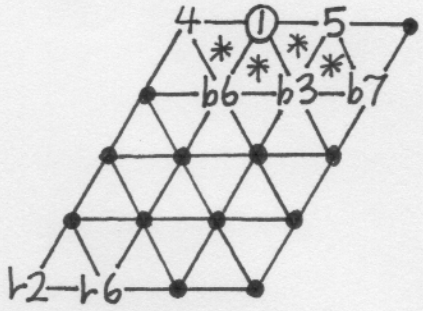
1	b2	b2	b3	4	5	b6	b7	– Phr. + b2
7	1	♯1	2	3	♯4	5	6	– Lyd. + ♯1
b7	b1	1	b2	b3	♯4	b5	b6	–
6	b7	b7	1	2	3	4	5	– Mix. + b7
5	b6	b6	b7	1	2	b3	4	– Aeo. + b6
4	b5	b5	b6	b7	1	b2	b3	– Loc. + b5
3	4	♯4	5	6	7	1	2	– Ion. + ♯4
2	b3	b3	4	5	6	b7	1	– Dor. + b3



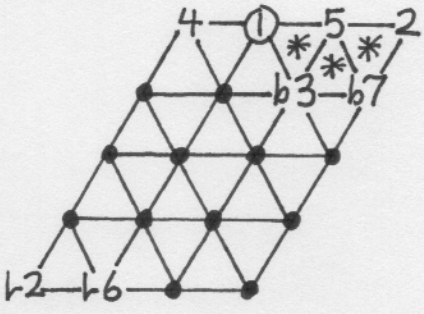
1	b2	2	b3	4	5	b6	b7	– Aeo. + b2
b7	1	♯1	b2	b3	♯4	b5	b6	–
b7	b1	1	b2	b3	4	b5	b6	– Loc. + b1
6	b7	7	1	2	3	4	5	– Ion. + b7
5	b6	6	b7	1	2	b3	4	– Dor. + b6
4	b5	5	b6	b7	1	b2	b3	– Phr. + b5
3	♯4	#4	5	6	7	1	2	– Lyd. + ♯4
2	b3	3	4	5	6	b7	1	– Mix. + b3



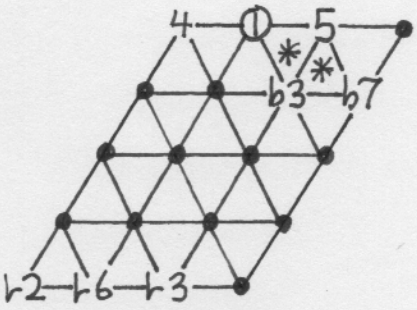
1	r2	b3	4	5	b6	r6	b7	– minor
r7	1	r2	r3	♯4	r5	5	r6	– neutral
6	r7	1	2	3	4	♯4	5	– major
5	r6	b7	1	2	b3	r3	4	– min.-neu.
4	r5	b6	b7	1	b2	r2	b3	–
3	♯4	5	6	7	1	♯1	2	– major
r3	4	r5	r6	r7	r1	1	r2	–
2	r3	4	5	6	b7	r7	1	– neutral



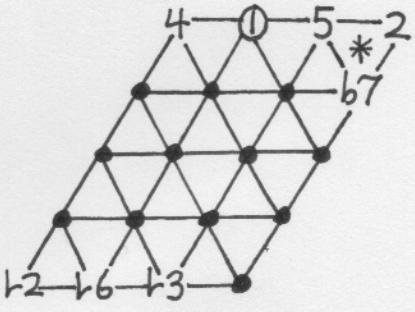
1	r2	2	b3	4	5	r6	b7	– minor
r7	1	♯1	r2	r3	♯4	5	r6	– neutral
b7	r1	1	b2	b3	4	r5	b6	–
6	r7	7	1	2	3	♯4	5	– major
5	r6	6	b7	1	2	r3	4	– neutral
4	r5	5	b6	b7	1	r2	b3	– minor
r3	4	♯4	r5	r6	r7	1	r2	–
2	r3	3	4	5	6	r7	1	– major



1	r2	b3	r3	4	5	r6	b7	– min.-neu.
r7	1	r2	2	r3	♯4	5	r6	– neutral
6	r7	1	♯1	2	3	♯4	5	– major
r6	b7	r1	1	r2	r3	4	r5	–
5	r6	b7	r7	1	2	r3	4	– neutral
4	r5	b6	r6	b7	1	r2	b3	–
r3	4	r5	5	r6	r7	1	r2	– neutral
2	r3	4	♯4	5	6	r7	1	– neutral



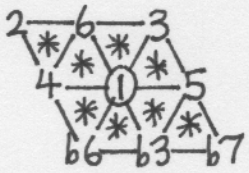
1	r2	2	r3	4	5	r6	b7	– neutral
r7	1	♯1	2	r3	♯4	5	r6	– neutral
b7	r1	1	r2	b3	4	r5	b6	–
r6	b7	r7	1	r2	r3	4	r5	–
5	r6	6	r7	1	2	r3	4	– neutral
4	r5	5	r6	b7	1	r2	b3	– minor
r3	4	♯4	5	r6	r7	1	r2	– neutral
2	r3	3	♯4	5	6	r7	1	– maj.-neu.

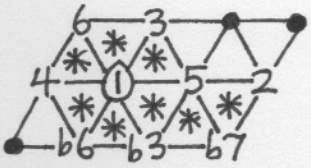


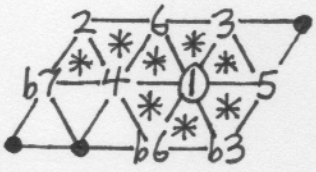
A few prominant higher Classes:

The Enneachord – symmetrical

1	2	b3	3	4	5	b6	6	b7 –
b7	1	b2	2	b3	4	b5	5	b6 –
6	7	1	#1	2	3	4	#4	5 –
b6	b7	b1	1	b2	b3	b4	4	b5 –
5	6	b7	7	1	2	b3	3	4 –
4	5	b6	6	b7	1	b2	2	b3 –
3	#4	5	#5	6	7	1	#1	2 –
b3	4	b5	5	b6	b7	b1	1	b2 –
2	3	4	#4	5	6	b7	7	1 –

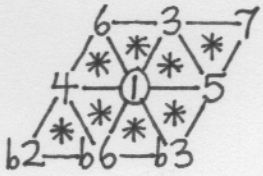






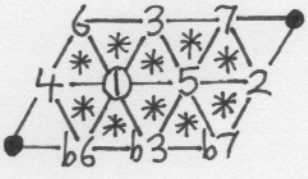
The Harmonic Ennead – Symmetrical
Core Chromatic (Septimal) pattern

1	b2	b3	3	4	5	b6	6	7 –
7	1	2	#2	3	#4	5	#5	#6 –
6	b7	1	#1	2	3	4	#4	#5 –
b6	bb7	b1	1	b2	b3	b4	4	5 –
5	b6	b7	7	1	2	b3	3	#4 –
4	b5	b6	6	b7	1	b2	2	3 –
3	4	5	#5	6	7	1	#1	#2 –
b3	b4	b5	5	b6	b7	b1	1	2 –
b2	bb3	b4	4	b5	b6	bb7	b7	1 –



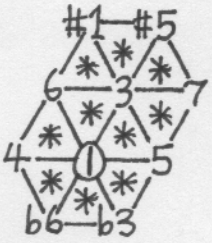
The Harmonic Decad – Symmetrical
Ellis' "Unit of Harmony"

1	2	b3	3	4	5	b6	6	b7	7 –
b7	1	b2	2	b3	4	b5	5	b6	6 –
6	7	1	#1	2	3	4	#4	5	#5 –
b6	b7	b1	1	b2	b3	b4	4	b5	5 –
5	6	b7	7	1	2	b3	3	4	#4 –
4	5	b6	6	b7	1	b2	2	b3	3 –
3	#4	5	#5	6	7	1	#1	2	#2 –
b3	4	b5	5	b6	b7	b1	1	b2	2 –
2	3	4	#4	5	6	b7	7	1	#1 –
b2	b3	b4	4	b5	b6	bb7	b7	b1	1 –



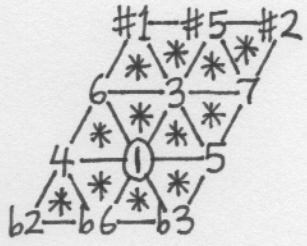
Core Enharmonic Decad – Symmetrical

1	#1	b3	3	4	5	#5	b6	6	7 –
b1	1	♯2	b3	b4	b5	5	♯5	b6	b7 –
6	#6	1	#1	2	3	#3	4	#4	#5 –
b6	6	b1	1	b2	b3	3	b4	4	5 –
5	#5	b7	7	1	2	#2	b3	3	#4 –
4	#4	b6	6	b7	1	#1	b2	2	3 –
b4	4	♯5	b6	♯6	♯7	1	♯1	b2	b3 –
3	#3	5	#5	6	7	#7	1	#1	#2 –
b3	3	b5	5	b6	b7	7	b1	1	2 –
b2	2	b4	4	b5	b6	6	♯6	b7	1 –



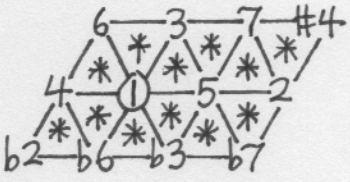
Enharmonic Dodecad – Symmetrical

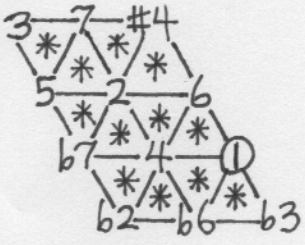
1	#1	b2	#2	b3	3	4	5	#5	b6	6	7 –
♯7	1	♯1	2	♯2	b3	b4	b5	5	♯5	b6	b7 –
7	♯7	1	♭2	2	#2	3	#4	♭5	5	#5	#6 –
♯6	b7	♭7	1	♯1	b2	♯2	b4	4	♯4	b5	b6 –
6	#6	b7	♭1	1	#1	2	3	♭4	4	#4	#5 –
b6	6	♯6	7	b1	1	b2	b3	3	b4	4	5 –
5	#5	b6	#6	b7	7	1	2	#2	b3	3	#4 –
4	#4	b5	#5	b6	6	b7	1	#1	b2	2	3 –
b4	4	♯4	5	♯5	b6	♯6	♯7	1	♯1	b2	b3 –
3	#3	4	♭5	5	#5	6	7	♭1	1	#1	#2 –
b3	3	b4	#4	b5	5	b6	b7	7	b1	1	2 –
b2	2	♯2	3	♯3	4	b5	b6	6	♯6	b7	1 –

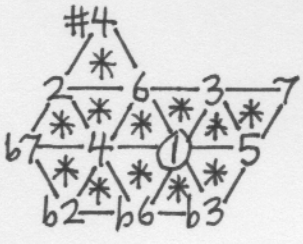


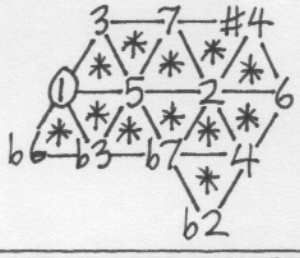
The Harmonic Dodecad – Symmetrical
The Full Chromatic Scale

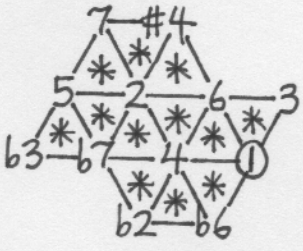
1	b2	2	b3	3	4	#4	5	b6	6	b7	7 –
7	1	#1	2	#2	3	#3	#4	5	#5	6	#6 –
b7	b1	1	b2	2	b3	3	4	b5	5	b6	6 –
6	b7	7	1	#1	2	#2	3	4	#4	5	#5 –
b6	bb7	b7	b1	1	b2	2	b3	b4	4	b5	5 –
5	b6	6	b7	7	1	#1	2	b3	3	4	#4 –
b5	♯5	b6	bb7	b7	b1	1	b2	bb3	b3	b4	4 –
4	b5	5	b6	6	b7	7	1	b2	2	b3	3 –
3	4	#4	5	#5	6	#6	7	1	#1	2	#2 –
b3	b4	4	b5	5	b6	6	b7	b1	1	b2	2 –
2	b3	3	4	#4	5	#5	6	b7	7	1	#1 –
b2	bb3	b3	b4	4	b5	5	b6	bb7	b7	b1	1 –





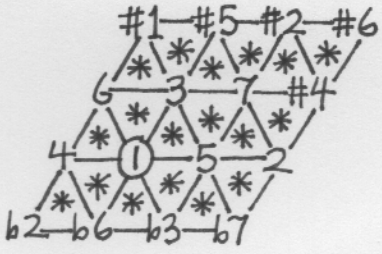






16-tone 'full' Enharmonic pattern:

1	#1	b2	2	#2	b3	3	4	#4	5	#5	b6	6	#6	b7	7 –
b1	1	♯1	b2	2	♯2	b3	b4	4	b5	5	♯5	b6	6	♯6	b7 –
7	#7	1	#1	♮2	2	#2	3	♮4	#4	♮5	5	#5	♮6	6	#6 –
b7	7	b1	1	#1	b2	2	b3	3	4	#4	b5	5	#5	b6	6 –
♯6	b7	♮7	b1	1	♯1	b2	♯2	b3	b4	4	♯4	b5	5	♯5	b6 –
6	#6	b7	7	♮1	1	#1	2	#2	3	♮4	4	#4	♮5	5	#5 –
b6	6	♯6	b7	7	b1	1	b2	2	b3	3	b4	4	#4	b5	5 –
5	#5	b6	6	#6	b7	7	1	#1	2	#2	b3	3	♮4	4	#4 –
b5	5	♯5	b6	6	♯6	b7	b1	1	b2	2	♯2	b3	3	b4	4 –
4	#4	b5	5	#5	b6	6	b7	7	1	#1	b2	2	#2	b3	3 –
b4	4	♯4	b5	5	♯5	b6	♯6	b7	b1	1	♯1	b2	2	♯2	b3 –
3	♮4	4	#4	♮5	5	#5	6	#6	7	♮1	1	#1	♮2	2	#2 –
b3	3	b4	4	#4	b5	5	b6	6	b7	7	b1	1	#1	b2	2 –
♯2	b3	♮3	b4	4	♯4	b5	♯5	b6	♯6	b7	♮7	b1	1	♯1	b2 –
2	#2	b3	3	#3	4	#4	5	#5	6	#6	b7	7	♮1	1	#1 –
b2	2	♯2	b3	3	b4	4	b5	5	b6	6	♯6	b7	7	b1	1 –

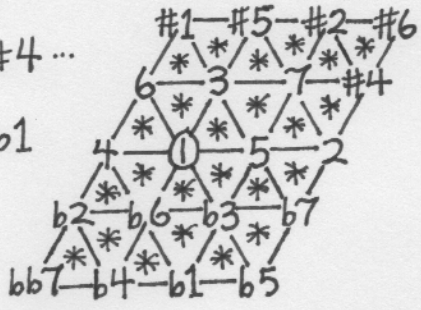


20-tone 'further expansion' pattern:

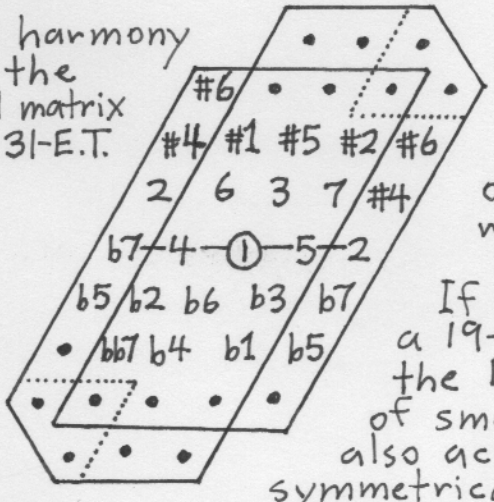
As an 'ultra-chromatic' scale

1 #1·b2 2 #2·b3 3·b4 4 #4…

·b5 5 #5·b6 6·♯6 bb7·#6·b7 7·b1



The harmony on the full matrix for 31-E.T.



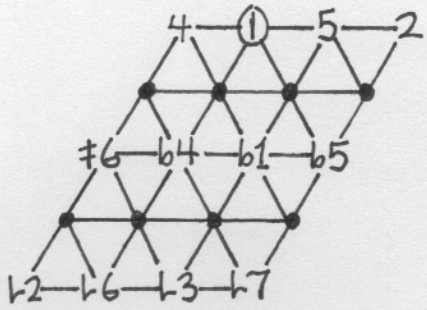
With wide expansion, the density of enharmonic relations increases, witness section 6·♯6·#6·b7.

If we omit the bb7 (♯6), then we have a 19-tone system approximation of the 19-Equal-Temperament — a scale of small (chromatic) semitones. We could also achieve this harmony in a mode symmetrical about the Tonic by omitting the #6 rather than the ♯6.

An Arabic-style Chromatic Scale – Symmetrical

It is symmetrical about an element not in the scale and included here in brackets :

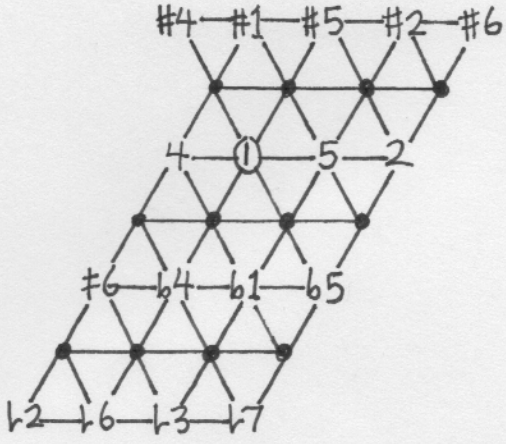
1	↓2	2	↓3	b4	4	b5	5	(#5)	↓6	♯6	↓7	b1 –
↓7	1	♯1	2	#2	↓3	↓4	♯4	(b5)	5	#5	6	#6 –
b7	↓1	1	↓2	♯2	b3	b4	4	(#4)	↓5	♯5	↓6	♯6 –
↓6	b7	↓7	1	#1	↓2	#2	↓3	(b4)	4	#4	5	#5 –
#5	♯6	#6	b1	1	#1	2	#2	(↓3)	b4	4	b5	5 –
5	↓6	6	↓7	b1	1	b2	2	(#2)	↓3	b4	♯4	b5 –
#4	♯5	#5	♯6	b7	7	1	#1	(↓2)	♯2	b3	b4	4 –
4	↓5	5	↓6	♯6	b7	b1	1	(#1)	↓2	♯2	↓3	b4 –
b4	#4	b5	#5	↓6	♯6	↓7	b1	(1)	#1	↓2	#2	↓3 –
↓3	4	♯4	5	#5	↓6	#6	↓7	(b1)	1	#1	2	#2 –
#2	b4	↓4	b5	5	#5	6	#6	(↓7)	b1	1	b2	2 –
↓2	b3	↓3	4	#4	↓5	#5	↓6	(♯6)	b7	7	1	#1 –
#1	♯2	#2	b4	4	#4	5	#5	(↓6)	♯6	b7	b1	1 –



A 17-tone extension of the above scale, also symmetrical about #5, forming a system approximation of the 19 equal-temperament, since most of the step intervals are chromatic semitones.

1 #1 ↓2 2 #2 ↓3 b4 4 #4 .....

b5 5 #5 ↓6 ♯6 #6 ↓7 b1



The Use of 'Tritone-Removed' Substitutions in 31-et

Diatonic Dom7 □	Chromatic Substitution	Resolution	Diatonic m6 □	Chromatic Substitution	Resolution
♭II7	♭VI h7	♭V (♭III m)	♭VI m6	bbIII m6 h	♭III m (♭V)
♭VI7	♭III h7	♭II (♭VII m)	♭III m6	bbVII m6 h	♭VII m (♭II)
♭III7	♭VII h7	♭VI (bbV m)	♭VII m6	bIV m6 h	bbV m (♭VI)
♭VII7	♭IV h7	♭III (bbII m)	bbV m6	bI m6 h	bbII m (♭III)
bbV7	♭I h7	♭VII (bbVI m)	bbII m6	bV m6 h	bbVI m (♭VII)
bbII7	♭V h7	bbV (bbIII m)	bbVI m6	bII m6 h	bbIII m (bbV)
bbVI7	♭II h7	bbII (bbVII m)	bbIII m6	bVI m6 h	bbVII m (bbII)
bbIII7	♭VI h7	bbVI (bIV m)	bbVII m6	bIII m6 h	bIV m (bbVI)
bbVII7	♭III h7	bbIII (bI m)	bIV m6	bVII m6 h	bI m (bbIII)
bIV7	♭VII h7	bbVII (bV m)	bI m6	IV m6 h	bV m (bbVII)
bI7	bbV h7	bIV (bII m)	bV m6	I m6 h	bII m (bIV)
bV7	bbII h7	bI (bVI m)	bII m6	V m6 h	bVI m (bI)
bII7	bbVI h7	bV (bIII m)	bVI m6	II m6 h	bIII m (bV)
bVI7	bbIII h7	bII (bVII m)	bIII m6	VI m6 h	bVII m (bII)
bIII7	bbVII h7	bVI (IV m)	bVII m6	III m6 h	IV m (bVI)
bVII7	bIV h7	bIII (I m)	IV m6	VII m6 h	I m (bIII)
IV7	bI h7	bVII (V m)	I m6	#IV m6 h	V m (bVII)
I7	bV h7	IV (II m)	V m6	#I m6 h	II m (IV)
* V7	bII h7	I (VI m)	II m6 □	#V m6 h	VI m (I)
II7	bVI h7	V (III m)	VI m6	#II m6 h	III m (V)
VI7	bIII h7	II (VII m)	III m6	#VI m6 h	VII m (II)
III7	bVII h7	VI (#IV m)	VII m6	#III m6 h	#IV m (VI)
VII7	IV h7	III (#I m)	#IV m6	#VII m6 h	#I m (III)
#IV7	I h7	VII (#V m)	#I m6	×IV m6 h	#V m (VII)
#I7	V h7	#IV (#II m)	#V m6	×I m6 h	#II m (#IV)
#V7	II h7	#I (#VI m)	#II m6	×V m6 h	#VI m (#I)
#II7	VI h7	#V (#III m)	#VI m6	×II m6 h	#III m (#V)
#VI7	III h7	#II (#VII m)	#III m6	×VI m6 h	#VII m (#II)
#III7	VII h7	#VI (×IV m)	#VII m	‡IV m6 h	×IV m (#VI)
#VII7	#IV h7	#III (×I m)	×IV m6	‡I m6 h	×I m (#III)
×IV7	#I h7	#VII (×V m)	×I m6	‡V m6 h	×V m (#VII)
×I7	#V h7	×IV (×II m)	×V m6	‡II m6 h	×II m (×IV)
×V7	#II h7	×I (×VI m)	×II m6	‡VI m6 h	×VI m (×I)
×II7	#VI h7	×V (‡IV m)	×VI m6	‡III m6 h	‡IV m (×V)
×VI7	#III h7	×II (‡I m)	‡IV m6	‡VII m6 h	‡I m (×II)
‡IV7	#VII h7	×VI (‡V m)	‡I m6	##IV m6 h	‡V m (×VI)
‡I7	×IV h7	‡IV (‡II m)	‡V m6	##I m6 h	‡II m (‡IV)