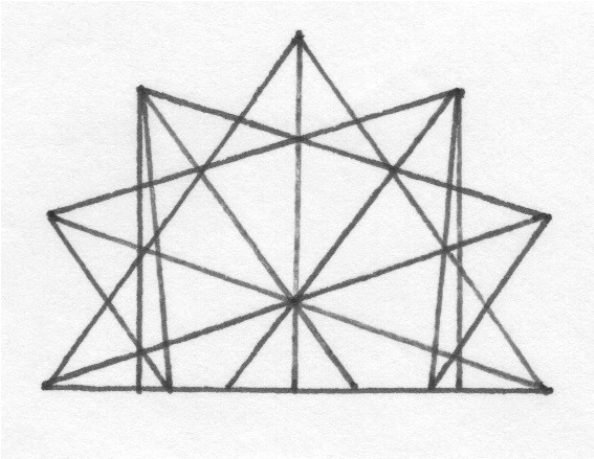
Notes on an Approach to Musical Composition in 31-E.T.
(and other Multiple Divisions) diagrams



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The **Hierarchy** of Musical Consonances (the Just ratios or frequency ratios of the Harmonic Series), showing the natural progression from consonance to dissonance:

1

Ratio [Harmonic Numbers]	Musical interval name - solfeggio - Functional name	
1:1	Unison - Tonic (I)	Perfect Consonances
2:1	Octave, 2nd Harmonic	
3:1	Musical twelfth - octave plus Fifth, 3rd Harmonic	
3:2	Perfect Fifth - Dominant (V)	
4:1	Double octave, 4th Harmonic	
4:3	Perfect Fourth - sub-dominant (IV)	
5:1	Octave plus major tenth, 5th Harmonic	Medial Consonances
5:2	Major tenth, octave plus major third	
5:3	Major sixth (VI)	

6

Senarius intervals
5-Limit

(Continued from previous page)

11:7 — Undecimal minor sixth (\b$\overline{\text{VI}}$) or sharp fifth (#$\overline{\text{V}}$)
11:8 — Double-diminished fifth (bb$\overline{\text{V}}$, ‡$\overline{\text{IV}}$), larger version
11:9 — Neutral third (r$\overline{\text{III}}$, x$\overline{\text{II}}$)
11:10 — Larger "three-quarter" tone (r$\overline{\text{II}}$, x$\overline{\text{I}}$)
12:1 — Triple octave plus fifth, 12th Harmonic
*12:5 — Just minor tenth
*12:7 — Septimal (semi-sharp) major sixth, Double-diminished seventh (‡$\overline{\text{VI}}$, bb$\overline{\text{VII}}$)
12:11 — Smaller "three-quarter" tone (r$\overline{\text{II}}$, x$\overline{\text{I}}$)

12

Duodecimal intervals
11-Limit

13:1 — Triple octave plus tridecimal minor sixth, 13th Harmonic
13:2 — Double octave plus tridecimal minor sixth
13:3 — Double octave plus tridecimal semitone
13:4 — Octave plus tridecimal minor sixth
13:5 — Octave plus sharp mediant
13:6 — Octave plus tridecimal semitone
13:7 — Tridecimal major seventh (\$\overline{\text{VII}}$)
13:8 — Tridecimal minor sixth, double-sharp fifth (/b$\overline{\text{VI}}$, x$\overline{\text{V}}$)
13:9 — Tridecimal diminished fifth (b$\overline{\text{V}}$)
13:10 — Sharp mediant (#$\overline{\text{III}}$)
13:11 — Tridecimal minor third (\b$\overline{\text{III}}$)
13:12 — Tridecimal semitone (/b$\overline{\text{II}}$, \x$\overline{\text{I}}$)

tridecimal dissonances

14:1 — Triple octave plus augmented sixth, 14th Harmonic
14:3 — Double octave plus subminor third
*14:5 — Octave plus augmented fourth, augmented eleventh
*14:9 — Septimal sharp (augmented) fifth (#$\overline{\text{V}}$)
14:11 — Undecimal major third or flat fourth (/$\overline{\text{III}}$, b$\overline{\text{IV}}$)
14:13 — Tridecimal semitone (/b$\overline{\text{II}}$)
15:1 — Triple octave plus major seventh, 15th Harmonic
15:2 — Double octave plus major seventh
15:4 — Octave plus major seventh
15:7 — Octave plus septimal diatonic (large) semitone
15:8 — Just major seventh ($\overline{\text{VII}}$)
15:11 — Undecimal double-diminished fifth, smaller version (bb$\overline{\text{V}}$, ‡$\overline{\text{IV}}$)
15:13 — Tridecimal double-diminished third (bb$\overline{\text{III}}$)
15:14 — Septimal diatonic (large) semitone (b$\overline{\text{II}}$)

Hard Dissonances

16:1 — Quadruple octave, 16th Harmonic
16:3 — Double octave plus a perfect Fourth
16:5 — Octave plus just minor sixth, minor thirteenth
*16:7 — Octave plus septimal wholetone
*16:9 — Pythagorean minor seventh (\b$\overline{\text{VII}}$)
16:11 — Undecimal double-augmented fourth (x$\overline{\text{IV}}$, r$\overline{\text{V}}$)
16:13 — Tridecimal neutral third (r$\overline{\text{III}}$)
16:15 — Just diatonic (large) semitone (b$\overline{\text{II}}$)

16

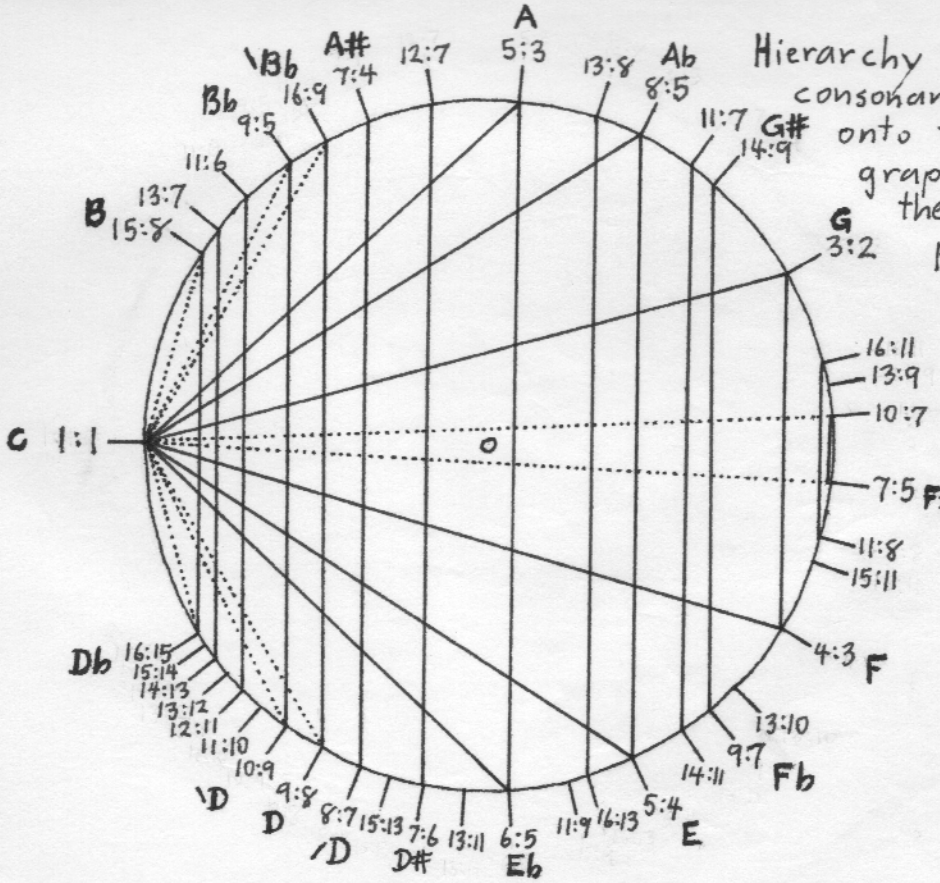
Hexadecimal intervals
13-Limit

The Hierarchy of Musical Consonances, employing the principle of Octave Equivalence, showing the natural progression from consonance to dissonance

Ratio	Musical interval name – solfeggio – Functional name		Cents
1:1	– Unison (I)		0
2:1	– Octave (I)	Perfect	1200
3:2	– Musical fifth (V)	Consonances	701.955
4:3	– Musical fourth (IV)		498.045
5:3	– Just major sixth (VI)	Medial	884.359
5:4	– Just major third (III)	Consonances	386.314
6:5	– Just minor third (bIII)	Imperfect consonances	315.641
7:4	– Augmented sixth, septimal minor seventh (#VI, hVII)		968.826
7:5	– Just septimal tritone, augmented Fourth (#IV)		582.512
7:6	– Augmented wholetone, subminor third (#II, hIII)		266.871
* 8:5	– Just minor sixth (bVI)		813.686
8:7	– Septimal wholetone (♯II, bbIII)		231.174
9:5	– Just minor seventh (bVII)	(♯III, bIV)	1017.596
9:7	– Septimal, dissonant major third, flattened fourth		435.084
9:8	– Major wholetone (II)	Soft	203.910
* 10:7	– septimal diminished fifth (bV)	Dissonances	617.488
10:9	– Minor wholetone (\II)		182.404
11:6	– Neutral seventh (hVII, xVI)	Neutral	1049.363
11:7	– Undecimal minor sixth (\bVI)	Dissonances	782.492
11:8	– Double-diminished fifth (bbV, ♯IV), larger version		551.318
11:9	– Neutral third (hIII, xII)		347.408
11:10	– Larger "three-quarter" tone (hII, xI)		165.004
* 12:7	– Double-diminished seventh, septimal major sixth (bbVII, ♯VI)		933.129
12:11	– Smaller "three-quarter" tone (hII, xI)		150.637
13:7	– Tridecimal major seventh (\VII)		1071.702
13:8	– Tridecimal minor sixth, double-sharp fifth (/bVI, xV, hVI)		840.528
13:9	– Tridecimal diminished fifth (bV)		636.618
13:10	– Sharp mediant (#III)	Tridecimal	454.214
13:11	– Tridecimal minor third (\bIII)	dissonances	289.210
13:12	– Tridecimal semitone (/bII, \xI)		138.573
* 14:9	– Septimal sharp (augmented) fifth (#V)		764.916
14:11	– Undecimal major third or flat fourth (/III, bIV)		417.508
14:13	– Tridecimal semitone (/bII)	Hard	128.298
15:8	– Just major seventh (VII)	Dissonances	1088.269
15:11	– Smaller undecimal double-diminished fifth (bbV, ♯IV)		536.951
15:13	– Tridecimal double-diminished third (bbIII)		247.741
15:14	– Septimal diatonic (large) semitone (bII)		119.443
* 16:9	– Pythagorean minor seventh (\bVII)		996.090
16:11	– Undecimal double augmented Fourth (xIV, hV)		648.682
16:13	– Tridecimal neutral third (hIII)		359.472
16:15	– Just diatonic (large) semitone (bII)		111.731

The Hierarchy of Musical Consonances, employing Octave Equivalence as well as the pairing of interval inversions, Pattern or Pitch Class:

1:1 and 2:1	— Unison (I) and Octave (I)	Perfect	Senarius intervals 5-Limit
3:2 and 4:3	— (V) and (IV)		
5:3 and 6:5	— (VI) and (bIII)] Medial Consonances		
5:4 and 8:5	— (III) and (bVI)		
7:4 and 8:7	— (#VI · bVII) and (♯II · bbIII)	Imperfect Consonances	
7:5 and 10:7	— (#IV · bV) and (##IV · bV)		
7:6 and 12:7	— (#II · bIII) and (♯VI · bbVII)		
9:5 and 10:9	— (bVII) and (\II)	Soft Dissonances	Tetractys intervals 7-Limit
9:7 and 14:9	— (♯III · bIV) and (#V · bVI)		
9:8 and 16:9	— (II) and (\bVII)		
11:6 and 12:11	— (rVII · xVI) and (rII · xI)	Neutral Dissonances	Duodecimal intervals 11-Limit
11:7 and 14:11	— (\bVI) and (/III)		
11:8 and 16:11	— (bbV · ♯IV) and (xIV · rV)		
11:9 and 18:11	— (rIII · xII) and (rVI · xV)		
11:10 and 20:11	— (rII · xI) and (rVII · xVI)		
13:7 and 14:13	— (\VII) and (/bII)	Tridecimal Dissonances	Hexadecimal intervals 13-Limit
13:8 and 16:13	— (xV) and (xII · rIII)		
13:9 and 18:13	— (bV) and (#IV)		
13:10 and 20:13	— (#III) and (bbVI)		
13:11 and 22:13	— (\bIII) and (/VI)		
13:12 and 24:13	— (/bII) and (\VII)		
15:8 and 16:15	— (VII) and (bII)	Hard Dissonances	
15:11 and 22:15	— (bbV · ♯IV) and (xIV · rV)		
15:13 and 26:15	— (bbIII) and (\#VI)		
15:14 and 28:15	— (bII) and (VII)		



Hierarchy of musical consonances projected onto the circular graph, showing their metrical properties.

The ratios are derived from the Harmonic Series numbers up to 16.

The Hierarchy of musical consonances applied to various multiple divisions, using the appropriate step numbers for each system, and employing octave equivalence

12-E.T.

0 - Unison
12 - Octave
7 - Fifth
5 - Fourth
9 - Major Sixth
4 - Major Third
3 - Minor Third
8 - Minor Sixth
10 - Minor Seventh
2 - Wholetone
11 - Major Seventh
1 - semitone
6 - Tritone

Perfect
Imperfect
Dissonant

19-E.T.

0 - Unison
19 - Octave
11 - Fifth
8 - Fourth
14 - Major Sixth
6 - Major Third
5 - Minor Third
9 - Augmented Fourth
13 - Minor sixth
16 - Minor seventh
7 - Diminished fourth
3 - Wholetone
10 - Diminished Fifth
12 - Augmented Fifth
17 - Major seventh
4 - Augmented second
2 - Diatonic semitone
18 - Augmented seventh
1 - chromatic semitone
15 - Augmented sixth

31-E.T.

I - 0 - Unison
I - 31 - Octave
V - 18 - Fifth
IV - 13 - Fourth
VI - 23 - Major sixth
III - 10 - Major third
bIII - 8 - Minor third
#VI - 25 - Augmented sixth
#IV - 15 - Augmented Fourth
#II · bIII - 7 - Sub-minor third
bVI - 21 - Minor sixth
#II - 6 - Septimal wholetone
bVII - 26 - Minor seventh
#III · bIV - 11 - Septimal major third
II - 5 - wholetone
bV - 16 - diminished Fifth
rVII - 27 - neutral seventh
#IV - 14 - semisharp Fourth
rIII - 9 - neutral third
rII - 4 - semiflat second
#VI · bbVII - 24 - Septimal major sixth
rVI - 22 - Semiflat sixth
#V - 20 - Augmented Fifth
VII - 28 - Major seventh
bII - 3 - Diatonic semitone
bI - 29 - Diminished tonic
#I - 2 - Chromatic semitone
#III - 12 - Augmented third
#V - 19 - "Wolf" semisharp Fifth
rV - 17 - Semiflat Fifth
#VII - 30 - sharp seventh
#I - 1 - enharmonic

Perfect Consonances
Imperfect Consonances
Soft Dissonances
Neutral Dissonances
Hard Dissonances

Pairing inversions (Pitch Class) in step numbers

19-E.T.

0 - 19
11 - 8
14 - 5
6 - 13
9 - 10
16 - 3
7 - 12
17 - 2
18 - 1
4 - 15

Perfect
Imperfect
Dissonance

31-E.T.

0 - 31
18 - 13
23 - 8
10 - 21
25 - 6
15 - 16
7 - 24
26 - 5
11 - 20
27 - 4
14 - 17
9 - 22
28 - 3
29 - 2
12 - 19
30 - 1

Perfect Consonances
Imperfect Consonances
Soft Dissonances
Neutral Dissonances
Hard Dissonances

12-E.T.

0 - 12
7 - 5
9 - 3
4 - 8
10 - 2
11 - 1
6

Perfect
Imperfect
Dissonance

Step numbers are herein used to indicate modal or Functional relations, "do re mi", as well as solfeggio when translated to singable syllables. Pitch is indicated in letters, eg. C#, A♭, G♯, E♭.

31-E.T.

expressed as an "ultra-chromatic" or step number scale showing the various notational analogues :

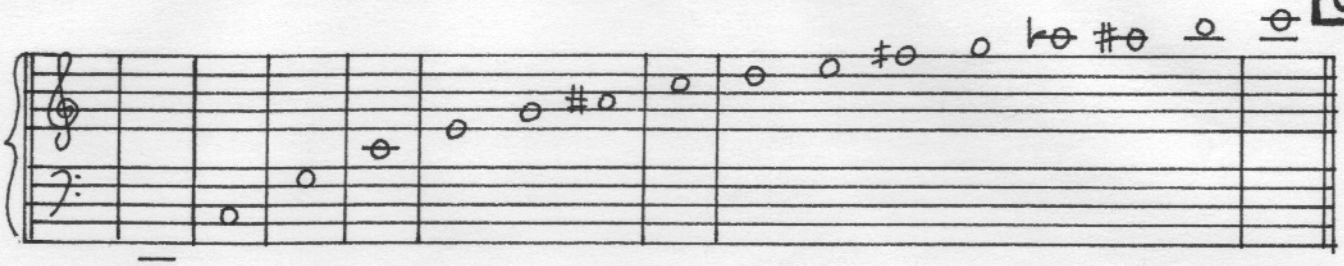
Step Number (as Function)	Function: Roman	Function: Modern	Solfeggio	Pitch (on C)	Step number (as Pitch)
31	I	1	do	C	31
30	#VII · ↓I	#7 · ↓1		B# · C↓	30
29	♯VII · bI	♯7 · b1		B♯ · Cb	29
28	VII	7	ti	B	28
27	xVI · ↓VII	x6 · ↓7		Ax · B↓	27
26	#VI · bVII	#6 · b7		A# · Bb	26
25	#VI · ↓VII	#6 · ↓7		A# · B↓	25
24	♯VI · bbVII	♯6 · bb7		A♯ · Bbb	24
23	VI	6	la	A	23
22	xV · ↓VI	x5 · ↓6		Gx · A↓	22
21	#V · bVI	#5 · b6		G# · Ab	21
20	#V · ↓VI	#5 · ↓6		G# · A↓	20
19	♯V · bbVI	♯5 · bb6		G♯ · Abb	19
18	V	5	so	G	18
17	xIV · ↓V	x4 · ↓5		Fx · G↓	17
16	#IV · bV	#4 · b5		F# · Gb	16
15	#IV · ↓V	#4 · ↓5		F# · G↓	15
14	♯IV · bbV	♯4 · bb5		F♯ · Gbb	14
13	IV	4	fa	F	13
12	#III · ↓IV	#3 · ↓4		E# · F↓	12
11	♯III · bIV	♯3 · b4		E♯ · Fb	11
10	III	3	mi	E	10
9	xII · ↓III	x2 · ↓3		Dx · E↓	9
8	#II · ↓III	#2 · b3		D# · Eb	8
7	#II · ↓III	#2 · ↓3		D# · E↓	7
6	♯II · bbIII	♯2 · bb3		D♯ · Ebb	6
5	II	2	re	D	5
4	xI · ↓II	x1 · ↓2		Cx · D↓	4
3	#I · bII	#1 · b2		C# · Db	3
2	#I · ↓II	#1 · ↓2		C# · D↓	2
1	♯I · bbII	♯1 · bb2		C♯ · Dbb	1
0	I	1	do	C	0

Notational signs after Fokker :

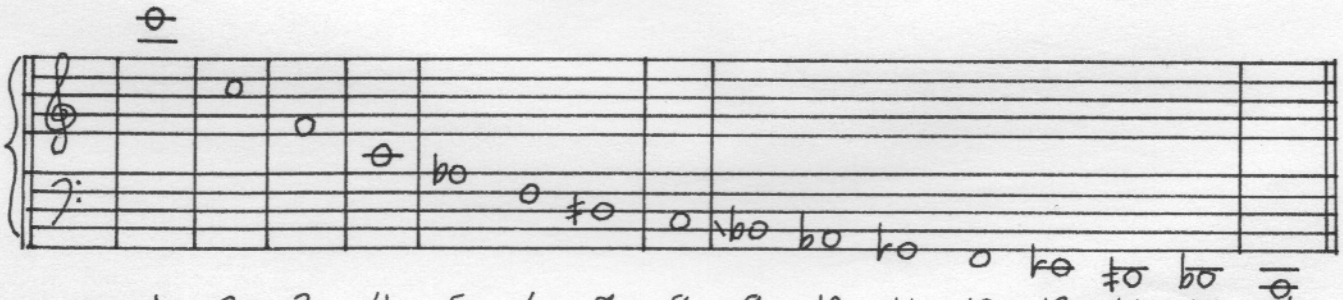
bb = double flat
↓ = sesqui flat
b = flat
↓ = semi flat
♯ = semisharp
= sharp
= sesquisharp
x = double sharp

The first 16 members of the Harmonic Series, and their notation: On Bass C. Shown below is the sub-harmonic series.

6



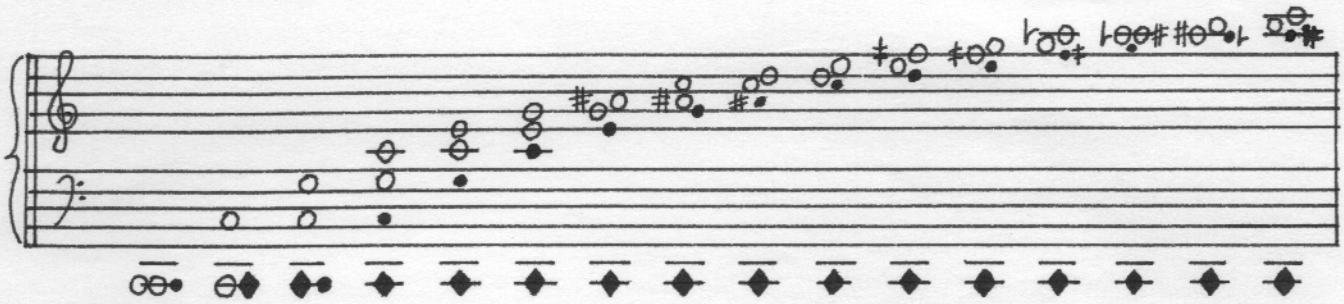
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	G	C	E	G	A# / B♭	C	D	E	G♭♭ / F♯	G	G× / A♭	A# / B♭	B	C



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	C	F	C	A♭	F	E♭♭ / D♯	C	B♭	A♭	F× / G♭	F	D× / E♭	E♭♭ / D♯	D♭	C

7

First and Second Order (◆ and •) Combination Tones (or also called Difference Tones) of Just Intervals classified according to position within the Harmonic Series (on Bass C). The series is arbitrarily limited to the 16th Harmonic. Harmonic numbers are shown below the Staff:



Interval	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
◆	—	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
•	1	—	1	2	3	4	5	6	7	8	9	10	11	12	13	14

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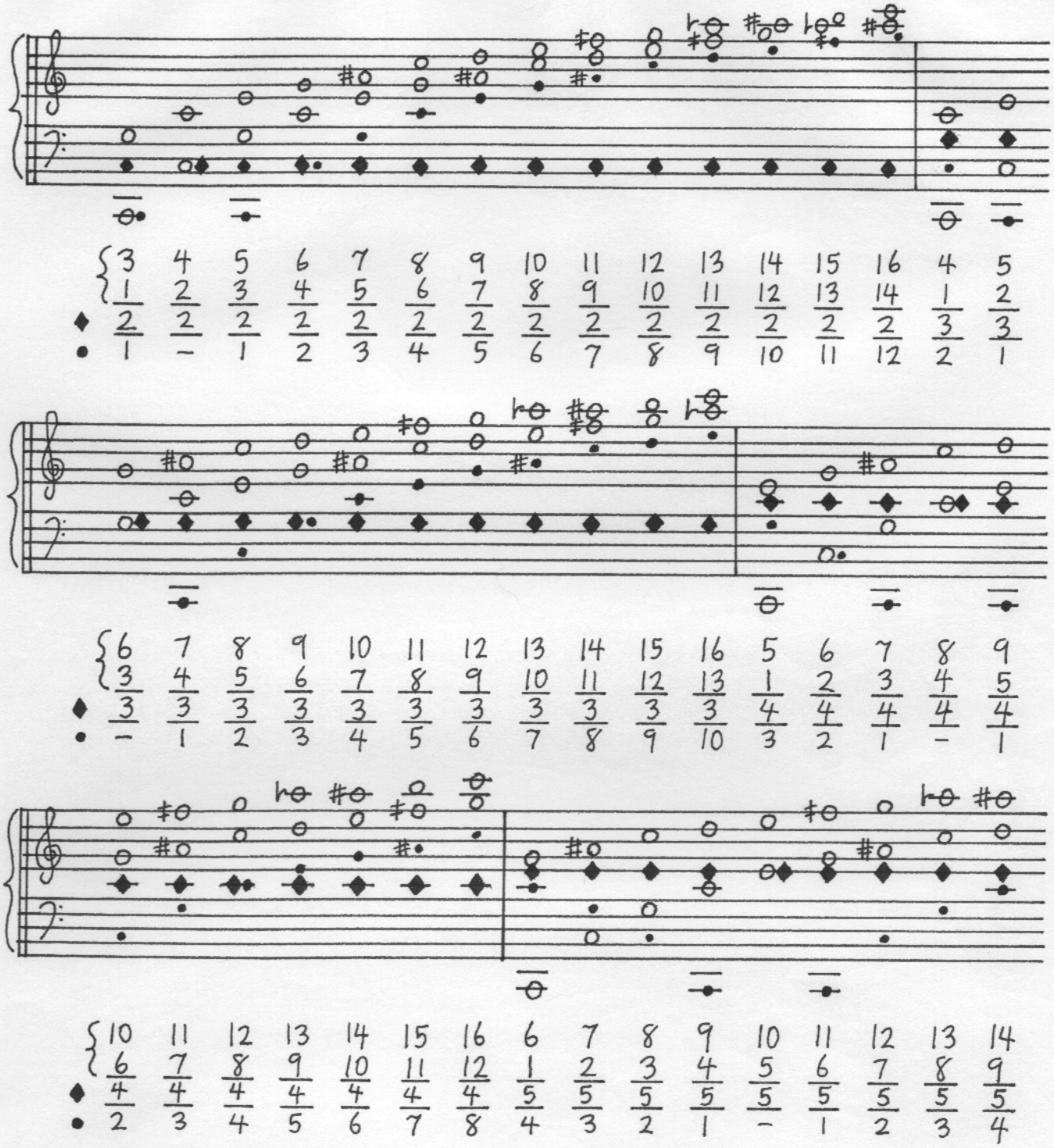

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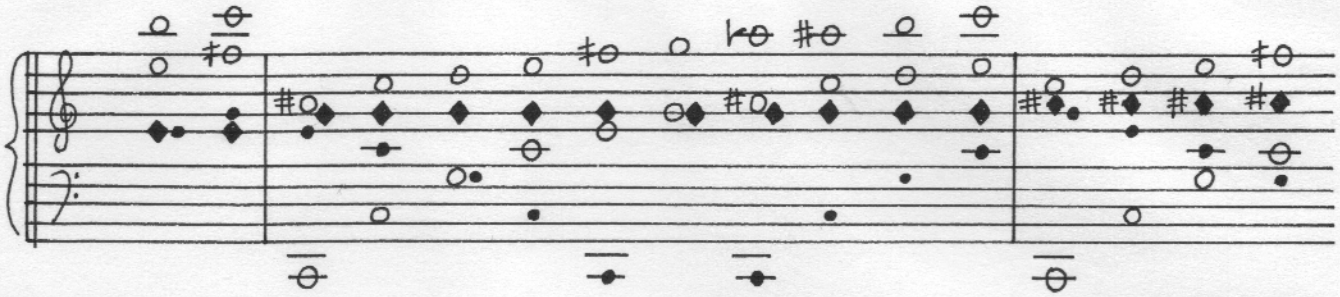
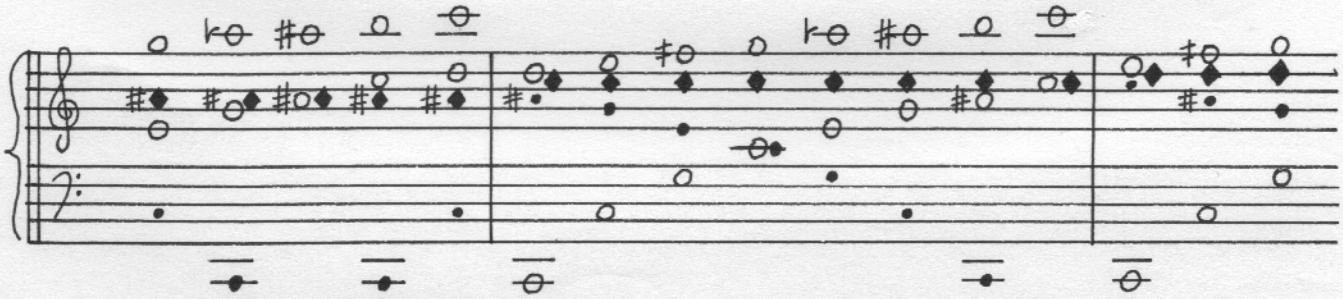
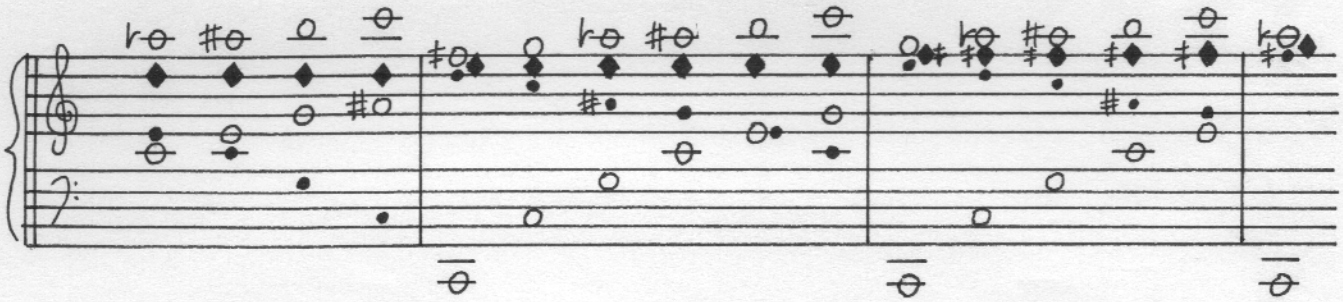


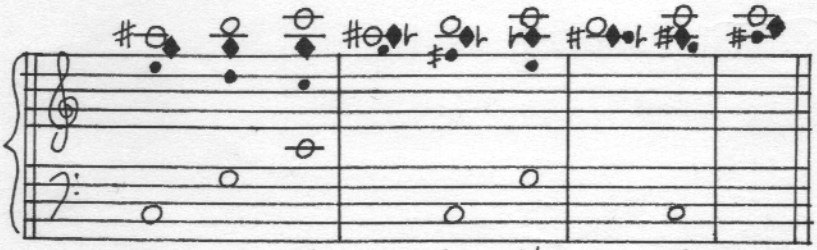
15	16	7	8	9	10	11	12	13	14	15	16	8	9	10	11
10	11	1	2	3	4	5	6	7	8	9	10	1	2	3	4
5	5	6	6	6	6	6	6	6	6	6	6	7	7	7	7
5	6	5	4	3	2	1	–	1	2	3	4	6	5	4	3



12	13	14	15	16	9	10	11	12	13	14	15	16	10	11	12
5	6	7	8	9	1	2	3	4	5	6	7	8	1	2	3
7	7	7	7	7	8	8	8	8	8	8	8	8	9	9	9
2	1	–	1	2	7	6	5	4	3	2	1	–	8	7	6



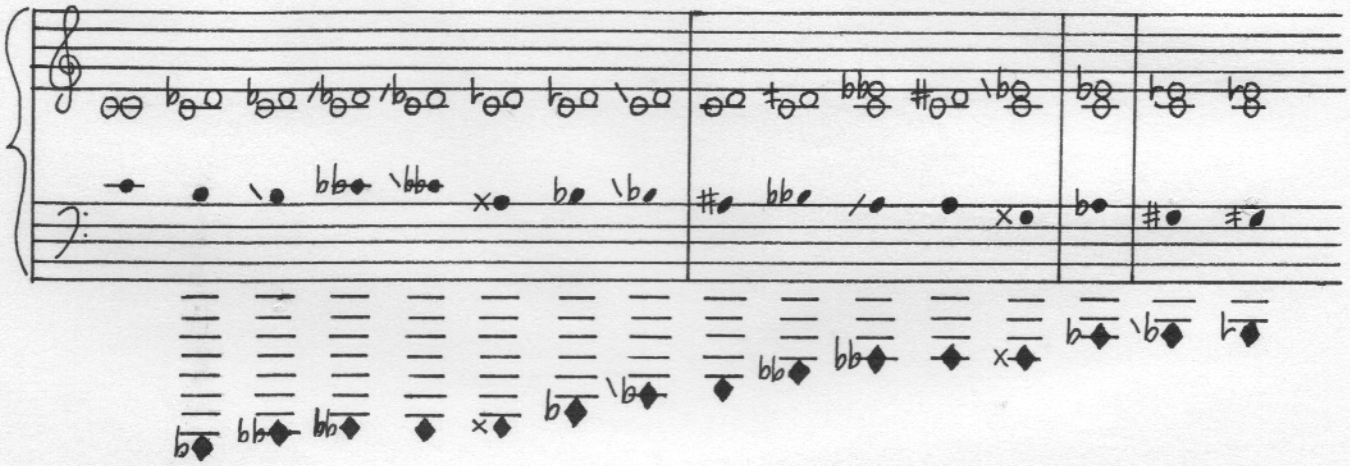
13	14	15	16	11	12	13	14	15	16	12	13	14	15	16	13
4	5	6	7	1	2	3	4	5	6	1	2	3	4	5	1
9	9	9	9	10	10	10	10	10	10	11	11	11	11	11	12
5	4	3	2	9	8	7	6	5	4	10	9	8	7	6	11



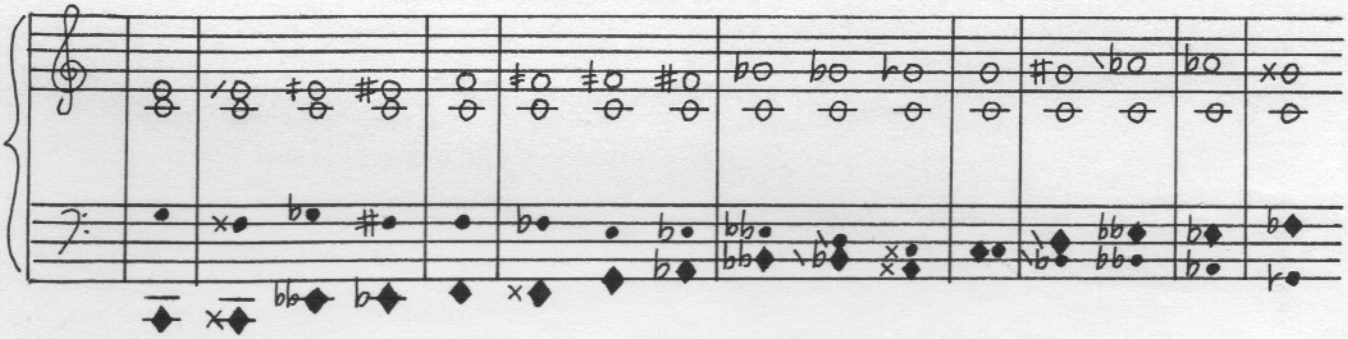
14	15	16		15	16		16	
2	3	4	14	2	3	15	2	16
12	12	12	1	13	13	1	14	1
10	9	8	13	11	10	14	12	15
			12			13		14

Combination tones of the first order (◆) and second order (•) for the Just interval series from unison to octave (C to C) using intervals from the Hierarchy (limit at 16):

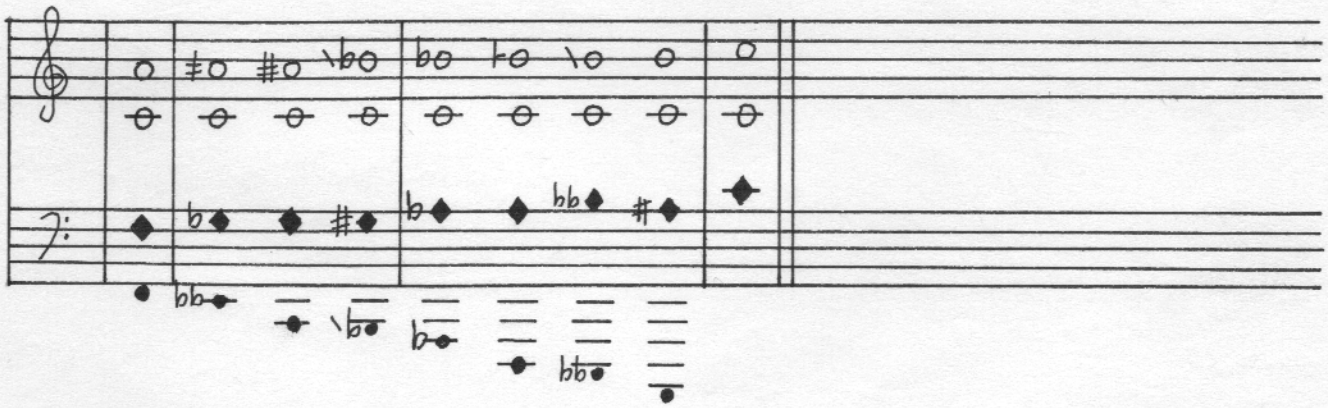
1:1 16:15 15:14 14:13 13:12 12:11 11:10 10:9 9:8 8:7 15:13 7:6 13:11 6:5 11:9 16:13



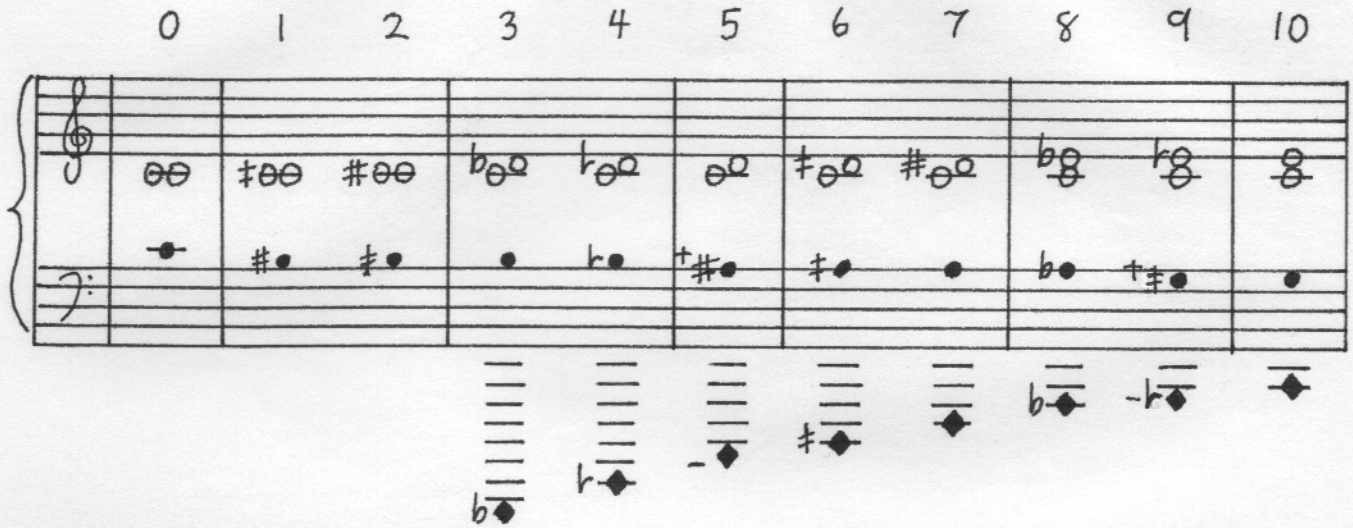
5:4 14:11 9:7 13:10 4:3 15:11 11:8 7:5 10:7 13:9 16:11 3:2 14:9 11:7 8:5 13:8

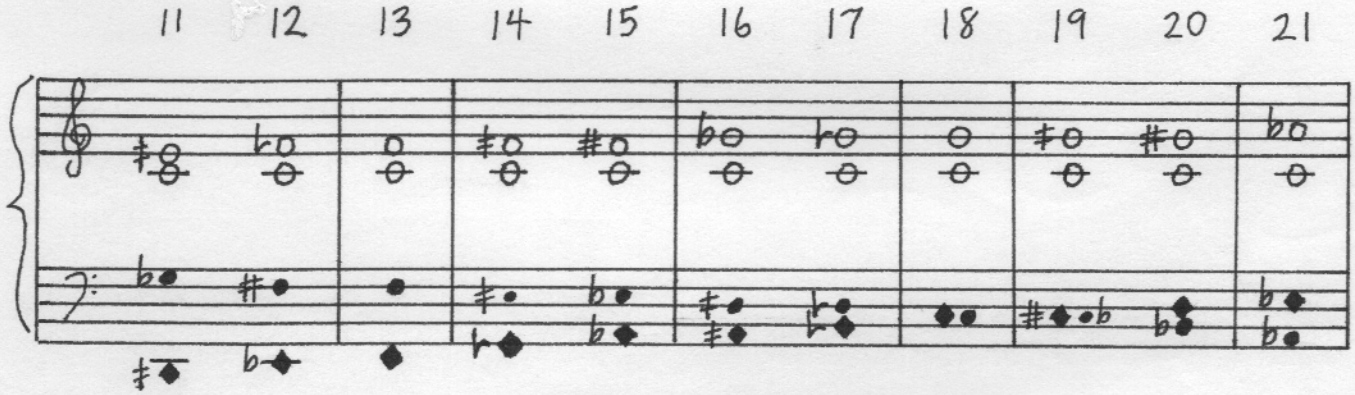


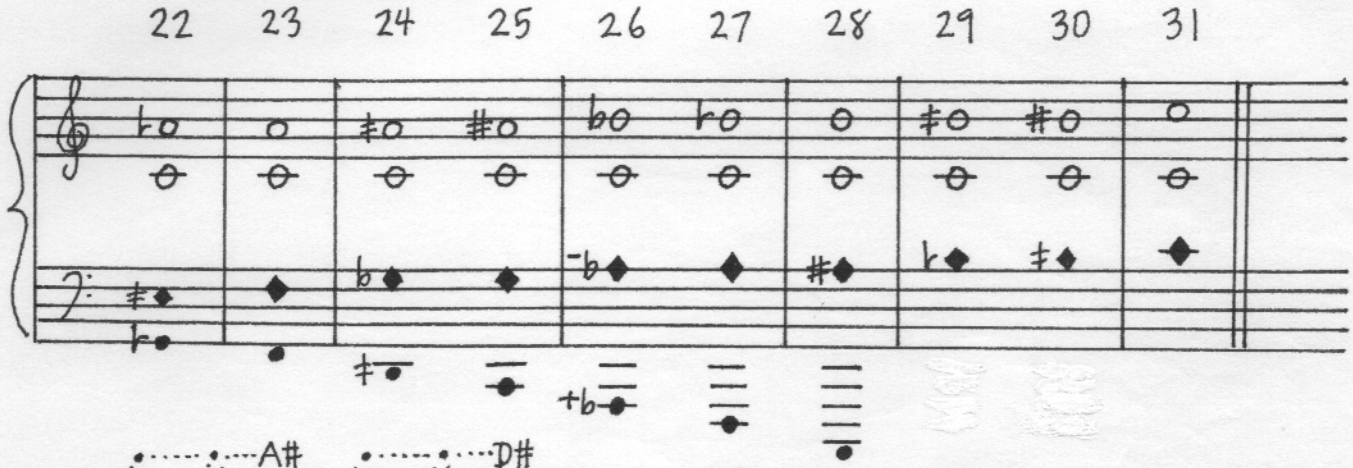
5:3 12:7 7:4 16:9 9:5 11:6 13:7 15:8 2:1

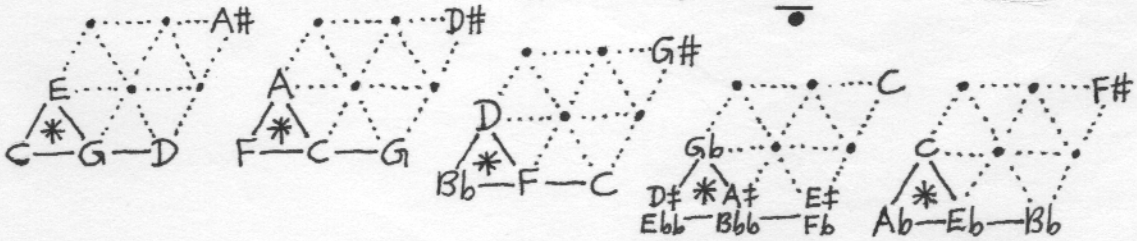


Combination tones of the first order (◆) and second order (•) applied to the enharmonic scale of 31-E.T.









The Just musical intervals (Frequency ratios) grouped according the progressive clouding of the inherent combination tones. The combination tones (1st and 2nd order difference tones) transform the interval into a sub-set chord of the Harmonic Series as indicated above the ratios:

← Perfect →		← Medial →	Imperfect		← Soft →		
C	C·G	C·G·E	C·G·A#	C·G·E·A#	C·E·D	C·A#·D	C·E·A#·D
1:1	3:1	5:1	7:3	7:1	9:4	9:1	9:2
2:1	3:2	5:2	7:4	7:2	9:5	9:8	9:7
	4:1	5:3	8:1	7:5	10:1	16:7	14:5
	4:3	5:4	8:7	7:6	10:9	16:9	14:9
		6:1		10:3			
		6:5		10:7			
		8:3		12:5			
		8:5		12:7			

← neutral →				← tridecimal →		
C·G·E·F♯	C·G·F♯·A#	C·E·D·F♯	C·A#·D·F♯	C·G·E·A♭	C·G·A#·A♭	C·G·F♯·A♭
11:3	11:4	11:1	11:2	13:8	13:6	13:1
11:5	11:7	11:10	11:9	16:3	13:7	13:5
11:6	14:3			16:13	13:10	13:12
11:8	14:11				14:1	
12:1					14:13	
12:11						
16:5						
16:11						

→			← hard →			
C·D·F♯·A♭	C·E·D·A♭	G·E·A#·A♭	C·B·A#	C·A#·F♯·B	C·F♯·A♭·B	C·A#·A♭·B
13:2	13:4	13:3	15:7	15:4	15:2	15:1
13:11	13:9		15:8	15:11	15:13	15:14
			16:1			
			16:15			

The same grouping as above but also employing octave equivalence and pairing of inversions:

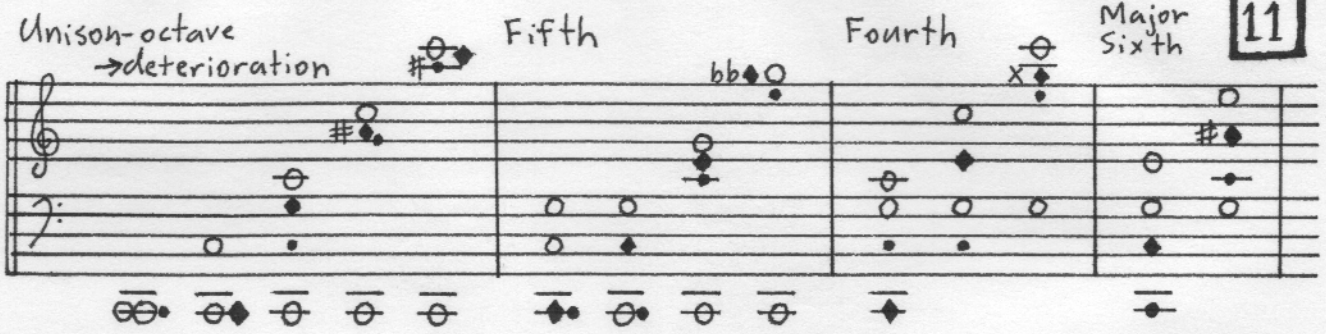
← Perfect →		Medial	Imperfect		← Soft →		
C	C·G	C·G·E	C·G·A#	C·G·E·A#	C·E·D	C·A#·D	C·E·A#·D
1:1–2:1	3:2–4:3	5:3–6:5	7:4–8:7	7:5–10:7	9:5–10:9	9:8–16:9	9:7–14:9
		5:4–8:5		7:6–12:7			

← neutral →				← tridecimal →	
C·G·E·F♯	C·G·A#·F♯	C·E·D·F♯	C·A#·D·F♯	C·G·E·A♭	C·G·A#·A♭
11:8–16:11	11:7–14:11	11:10–(20:11)	11:9–(18:11)	13:8–16:13	13:7–14:13
11:6–12:11					13:10–(20:13)

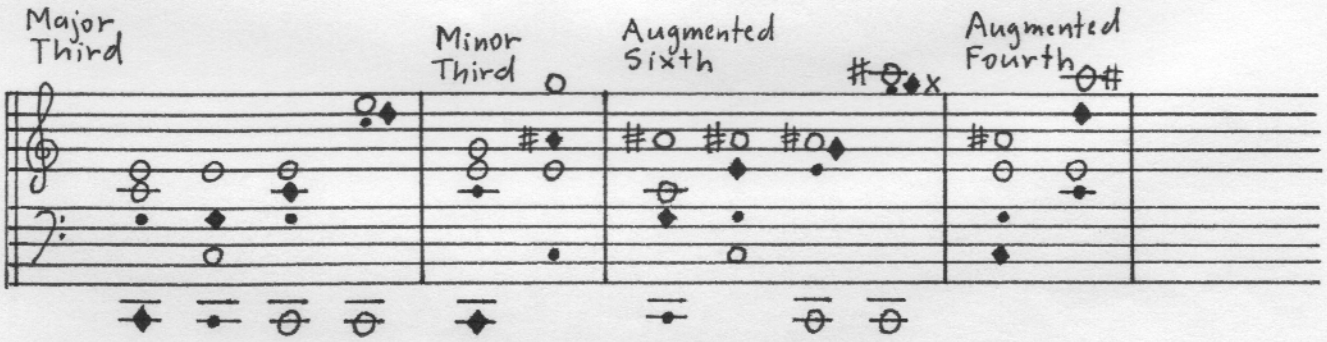
→			← hard →		
C·G·F♯·A♭	C·D·F♯·A♭	C·E·D·A♭	C·A#·B	C·A#·F♯·B	C·F♯·A♭·B
13:12–(24:13)	13:11–(22:13)	13:9–(18:13)	15:8–16:15	15:11–(22:15)	15:13–(26:15)

→
C·A#·A♭·B
15:14–(28:15)

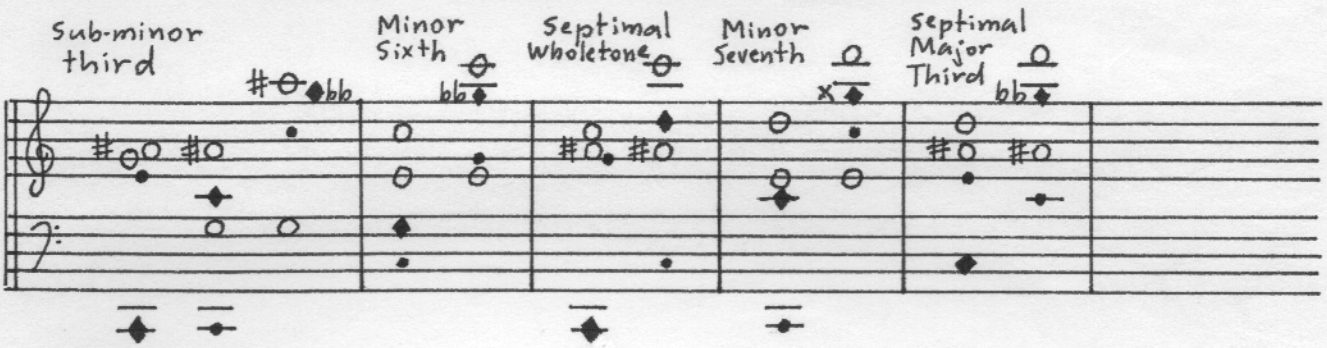
Consonance-altering effects due to the compounding of musical intervals, showing the influence of 1st + 2nd order Combination tones:



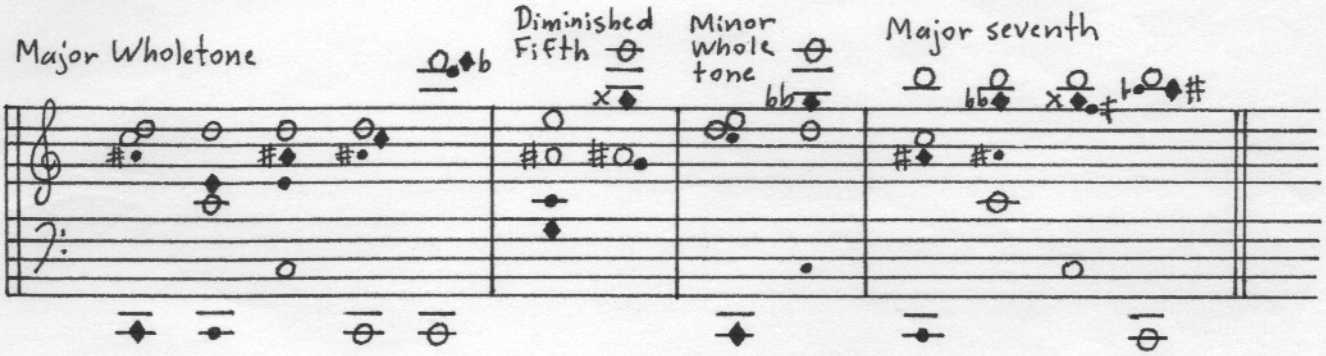
1:1 2:1 4:1 8:1 16:1 3:2 3:1 6:1 12:1 4:3 8:3 16:3 5:3 10:3



5:4 5:2 5:1 10:1 6:5 12:5 7:4 7:2 7:1 14:1 7:5 14:5



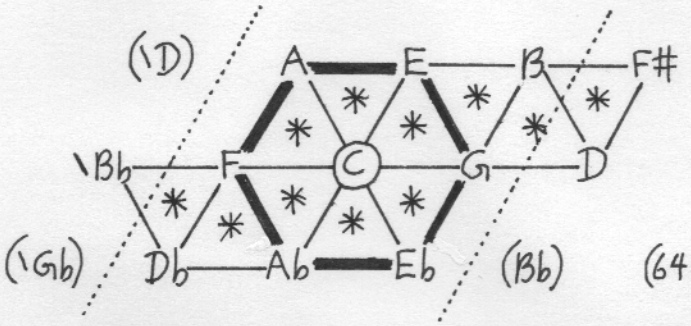
7:6 7:3 14:3 8:5 16:5 8:7 16:7 9:5 18:5 9:7 18:7



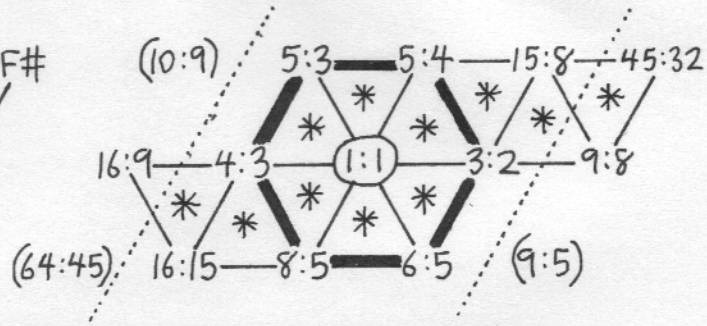
9:8 9:4 9:2 9:1 18:1 10:7 20:7 10:9 20:9 15:8 15:4 15:2 15:1

Hindemith's Just Chromatic Scale: Graphed on the Matrix of 3x5 Just Intonation

Pitch Notation (on C)



Ratio Notation (Just norms)



The comma-altered notation is not Hindemith's, neither is the expression as a just intonation matrix. Using Hindemith's "family" analogy for the interval "births" (page 39), the "sons" G, F, E, A, Eb and Ab coincide with Ellis' Harmonic Heptad. It is highlighted as the central hexagon of consonance. The remaining tones are "grandsons": D, \Bb, B, Db, and F# since they are generated by the "sons" and not the father "C."

Fibonacci-type relations among the intervals of the multiple division intonation systems 7, 12, 19, 31, 50, 81, 131, 212 – these systems themselves relate in a Fibonacci-type manner; for example, 19=7+12, 31=12+19, 50=19+31 etc. Only the first five are of musical interest, except perhaps 212-E.T since it is 4 × 53-E.T. If the Fibonacci series 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55...etc is translated to the intervals below, the graph shows the step number to achieve the desired interval.

System	Interval: Comma	Diesis	Chromatic Semitone	Diatonic Semitone	Whole-Tone	Minor Third	Perfect Fourth	Minor Sixth	Minor Ninth
7-E.T.	–	–	–	–	1	2	3	5	8
12-E.T.	–	–	–	1	2	3	5	8	13
19-E.T.	–	–	1	2	3	5	8	13	21
31-E.T.	–	1	2	3	5	8	13	21	34
50-E.T.	1	2	3	5	8	13	21	34	55
81-E.T.	2	3	5	8	13	21	34	55	89
131-E.T.	3	5	8	13	21	34	55	89	144
212-E.T.	5	8	13	21	34	55	89	144	233

Deviation of several prominant equal division intonations from the Just Intonation norms (the hierarchy of consonance, using the principle of octave equivalence). All measurements are in Cents and/or system step numbers where appropriate. The judgement is based on the rule of thumb: 0 to 2 cents – excellant, 2 to 6 cents – good, 6 to 12 cents – useable, 12 to 18 cents – mediocre, 18 to 24 cents – poor, 24 cents up – very poor.

Ratio	12-E.T. closest step	12-E.T. Cents	12-E.T. Judgement	19-E.T.	19-E.T.	19-E.T.	31-E.T.	31-E.T.	31-E.T.	53-E.T.	53-E.T.	53-E.T.
1:1	0	0	Perfect	0	0	Perfect	0	0	Perfect	0	0	Perfect
2:1	0	0	Perfect	0	0	Perfect	0	0	Perfect	0	0	Perfect
3:2	7	-1.955	excellant	11	-7.218	useable	18	-5.181	good	31	-0.068	excellant
4:3	5	+1.955	excellant	8	+7.218	useable	13	+5.181	good	22	+0.068	excellant
5:3	9	+15.641	mediocre	14	-0.149	excellant	23	+5.964	good	39	-1.340	excellant
5:4	4	+13.686	mediocre	6	-7.367	useable	10	+0.783	excellant	17	-1.408	excellant
6:5	3	-15.641	mediocre	5	+0.149	excellant	8	-5.964	good	14	+1.340	excellant
7:4	10	+31.174	very poor	15	-21.458	poor	25	-1.084	excellant	43	+4.759	good
7:5	6	+17.488	mediocre	9	-14.091	mediocre	15	-1.867	excellant	26	+6.167	useable
7:6	3	+33.129	very poor	4	-14.239	mediocre	7	+4.097	good	12	+4.827	good
8:5	8	-13.686	mediocre	13	+7.367	useable	21	-0.783	excellant	36	+1.408	excellant
8:7	2	-31.174	very poor	4	+21.458	poor	6	+1.084	excellant	10	-4.759	good
9:5	10	-17.596	mediocre	16	-7.070	useable	26	-11.144	useable	45	+1.272	excellant
9:7	4	-35.084	very poor	7	+7.021	useable	11	-9.278	useable	19	-4.895	good
9:8	2	-3.910	good	3	-14.436	mediocre	5	-10.361	useable	9	-0.137	excellant
10:7	6	-17.488	mediocre	10	+14.091	mediocre	16	+1.867	excellant	27	-6.167	useable
10:9	2	+17.596	mediocre	3	+7.070	useable	5	+11.144	mediocre	8	-1.272	excellant
11:6	10	-49.363	very poor	17	+24.321	very poor	27	-4.202	good	46	-7.854	useable
11:7	8	+17.508	mediocre	12	-38.561	very poor	20	-8.298	useable	35	+9.961	useable
11:8	6	+48.682	very poor	9	+17.103	mediocre	14	-9.383	useable	24	-7.922	useable
11:9	3	-47.408	very poor	6	+46.055	very poor	9	+0.979	excellant	15	-7.785	useable
11:10	2	+34.996	very poor	3	+24.470	very poor	4	-10.165	useable	7	-6.513	useable
12:7	9	-33.129	very poor	15	+14.239	mediocre	24	-4.097	good	41	-4.827	good
12:11	2	+49.363	very poor	2	-24.321	very poor	4	+4.202	good	7	+7.854	useable
13:7	11	+28.298	very poor	17	+1.982	excellant	28	+12.169	mediocre	47	-7.551	useable
13:8	8	-40.528	very poor	13	-19.475	poor	22	+11.085	useable	37	-2.792	good
13:9	6	-36.618	very poor	10	-5.039	good	16	-17.263	mediocre	28	-2.656	good
13:10	5	+45.786	very poor	7	-12.109	mediocre	12	+10.302	useable	20	-1.384	excellant
13:11	3	+10.790	useable	5	+26.579	very poor	7	-18.242	poor	13	+5.130	good
13:12	1	-38.573	very poor	2	-12.257	mediocre	4	+16.266	mediocre	6	-2.724	good
14:9	8	+35.084	very poor	12	-7.021	useable	20	-9.278	useable	34	+4.895	good
14:11	4	-17.508	mediocre	7	+38.561	very poor	11	-8.298	useable	18	-9.961	useable
14:13	1	-28.298	very poor	2	-1.982	excellant	3	-12.169	mediocre	6	+7.551	useable
15:8	11	+11.731	useable	17	-14.585	mediocre	28	-4.398	good	48	-1.476	excellant
15:11	5	-36.951	very poor	9	-31.470	very poor	14	+4.984	good	24	+6.445	useable
15:13	2	-47.741	very poor	4	+4.891	good	6	-15.483	mediocre	11	+1.316	excellant
15:14	1	-19.443	poor	2	+6.873	useable	3	-3.314	good	5	-6.235	useable
16:9	10	+3.910	good	16	+14.436	mediocre	26	+10.361	useable	44	+0.137	excellant
16:11	6	-48.682	very poor	10	-17.103	mediocre	17	+9.383	useable	29	+7.922	useable
16:13	4	+40.528	very poor	6	+19.475	poor	9	-11.085	useable	16	+2.792	good
16:15	1	-11.731	useable	2	+14.585	mediocre	3	+4.398	good	5	+1.476	excellant

The Triad Classes of 31-E.T.

with one example per class, using step number notation. Some of them are given additional names and are part of my inventory of triads. The 145 classes are first grouped into symmetrical and non-symmetrical patterns. The latter pair by compliments. In 31-E.T. every class has 31 member patterns, so that the total number of triad patterns is 145×31=4495.

Symmetrical Classes (15)

0, 1, 2
0, 2, 4
0, 3, 6
0, 4, 8
0, 5, 10 - "9Δ"
0, 6, 12
0, 7, 14
0, 8, 16 - "°Δ" - diminished Δ
0, 9, 18 - neutral Δ
0, 10, 20 - "+Δ" - augmented Δ
0, 11, 22
0, 12, 24
0, 13, 26 - "susΔ" - suspended Δ
0, 14, 28
0, 15, 30

Non-symmetrical Classes (130) presented as 65 pairs of compliments

0,1,3 - "Enh.Δ②"	0,2,3 - "Enh.Δ①"
0,1,4	0,3,4
0,1,5	0,4,5
0,2,5 - "ChromΔ①"	0,3,5 - "ChromΔ②"
0,1,6	0,5,6
0,2,6	0,4,6
0,1,7	0,6,7
0,2,7	0,5,7
0,3,7	0,4,7
0,1,8	0,7,8
0,2,8	0,6,8
0,3,8 - "mb9scalarΔ"	0,5,8 - "m9scalarΔ"
0,1,9	0,8,9
0,2,9	0,7,9
0,3,9	0,6,9
0,4,9	0,5,9
0,1,10	0,9,10
0,2,10 - "Mb9Δ"	0,8,10 - "MmΔ"
0,3,10 - "Sept. Mb9Δ"	0,7,10 - "Sept. MmΔ"
0,4,10	0,6,10
0,1,11	0,10,11

(continued on next column)

0,2,11	0,9,11
0,3,11 - "Diss. Mb9Δ"	0,8,11 - "Diss. MmΔ"
0,4,11	0,7,11
0,5,11 - "Sept. 9Δ②"	0,6,11 - "Sept. 9Δ①"
0,1,12	0,11,12
0,2,12	0,10,12
0,3,12	0,9,12
0,4,12	0,8,12
0,5,12	0,7,12
0,1,13	0,12,13
0,2,13 - "Sept. m6Δ"	0,11,13 - "Sept. M7Δ"
0,3,13 - "m6Δ"	0,10,13 - "M7Δ"
0,4,13	0,9,13
0,5,13 - "6Δ"	0,8,13 - "7Δ"
0,6,13 - "Sept. 6Δ"	0,7,13 - "Sept. 7Δ"
0,1,14	0,13,14
0,2,14	0,12,14
0,3,14	0,11,14
0,4,14	0,10,14
0,5,14	0,9,14
0,6,14	0,8,14
0,1,15	0,14,15
0,2,15 - "Sept. 5b5Δ"	0,13,15 - "Sept. 5b9Δ"
0,3,15	0,12,15
0,4,15	0,11,15
0,5,15 - "Dom7Δ"	0,10,15 - "Mb5Δ"
0,6,15	0,9,15
0,7,15 - "Sept. °Δ②"	0,8,15 - "Sept. °Δ①"
0,2,16	0,14,16
0,3,16 - "5b5Δ"	0,13,16 - "5b9Δ"
0,4,16	0,12,16
0,5,16 - "Diss. Dom7Δ"	0,11,16 - "Diss. Mb5Δ"
0,6,16 - "Sept. Dom7Δ"	0,10,16 - "Sept. Mb5Δ"
0,7,16	0,9,16
0,4,17	0,13,17
0,5,17	0,12,17
0,6,17	0,11,17
0,7,17	0,10,17
0,8,17	0,9,17
0,6,18 - "Sept. SusΔ①"	0,12,18 - "Sept. SusΔ②"
0,7,18 - "Subminor Δ"	0,11,18 - "Diss. MΔ"
0,8,18 - "mΔ"	0,10,18 - "MΔ"
0,8,19 - "Wolf mΔ"	0,11,19 - "Wolf MΔ"
0,9,19	0,10,19

An abbreviated inventory of Harmonic pattern CLASSES

extracted from the dictionary of 31-E.T. patterns minus the circular graph and Table of Functions indicating class name and Field (matrix) pattern

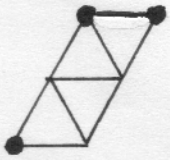
Intervals – 16 Classes shown on Figure 4

Triads

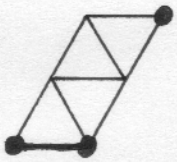
Major Triad (MΔ)
0,10,18

Minor Triad (mΔ)
0,8,18

Septimal Major Triad
Supra-major Triad
0,11,18

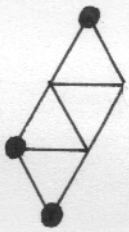


Septimal Minor Triad
Sub-minor triad
0,7,18



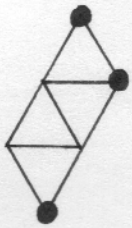
Wolf Major Triad

0,11,19



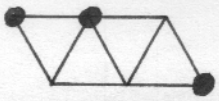
Wolf Minor Triad

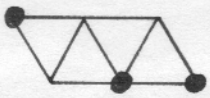
0,8,19



Suspended Triad (SusΔ) – symmetrical

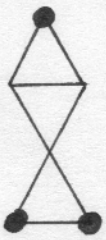
0,13,18
18,0,5



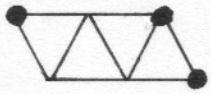


Septimal Suspended Δ①
0,13,19
18,0,6

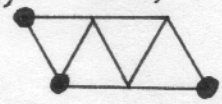
Septimal Suspended Δ②
0,12,18
19,0,6



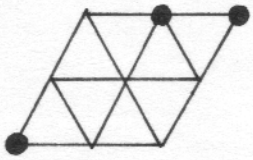
The Six Triad (6Δ) 0,18,23



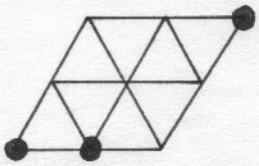
The Seventh Triad (7Δ) 0,18,26



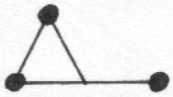
Septimal Six Δ
0,18,24

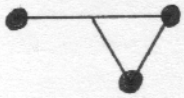


Septimal Seven Δ
0,18,25

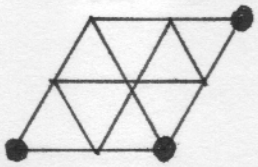


The Nine Triad (9Δ) – Symmetrical 0,5,10

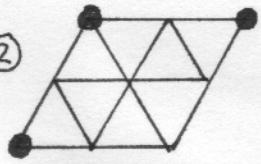




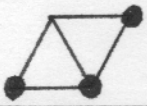
Septimal Nine Δ ①
0, 5, 25



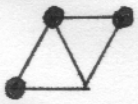
Septimal Nine Triad ②
0, 6, 26



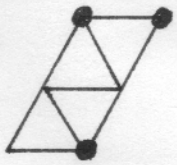
Major Seven Triad (M7Δ)
0, 18, 28



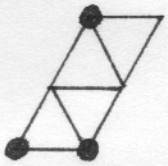
Minor Sixth Triad (m6Δ)
0, 18, 21



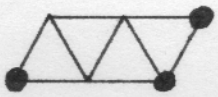
Septimal Major Seven Δ
0, 18, 29

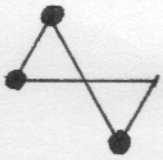


Septimal Minor Sixth Δ
0, 18, 20

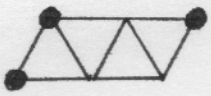


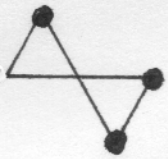
Dominant Seven Δ (Dom 7Δ) 0, 10, 26



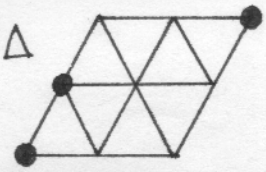


Major Flat Five Δ (Mb5Δ) 0, 10, 15

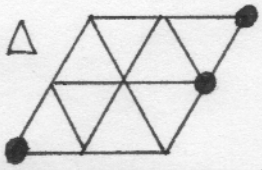




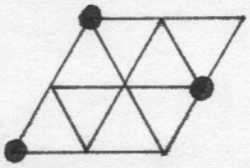
Septimal Dominant 7Δ
0, 10, 25

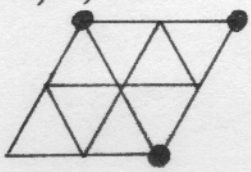


Septimal Major Flat-Five Δ
0, 10, 16

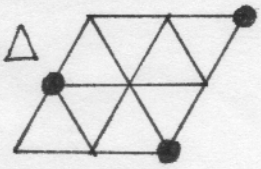


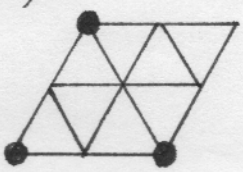
Dissonant Dom 7Δ
0, 11, 26

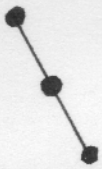




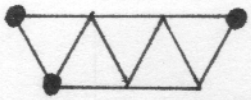
Dissonant Major Flat Five Δ
0, 11, 16

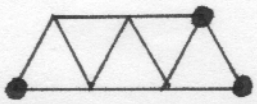




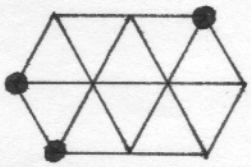


Diminished Triad (°Δ) - Symmetrical 0, 8, 23

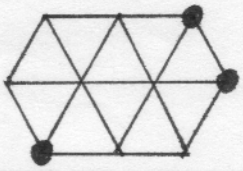




Septimal °Δ ①
0, 7, 23

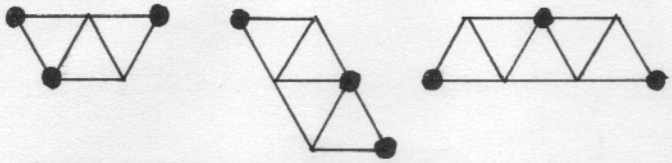


Septimal °Δ ②
0, 8, 24

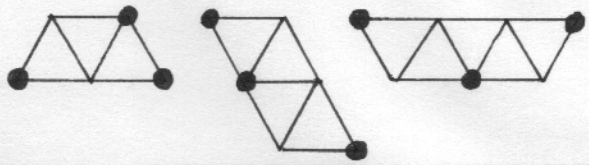


Augmented Triad (+Δ) - Symmetrical 0, 10, 21

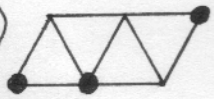
Minor Nine Scalar Δ (m9Δ) 0,5,8



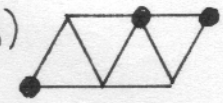
Minor Flat-nine Scalar Δ (mb9Δ) 0,3,8



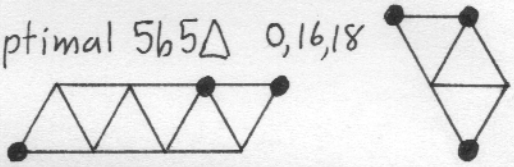
Five Flat-Five Triad (5b5Δ)
0,15,18



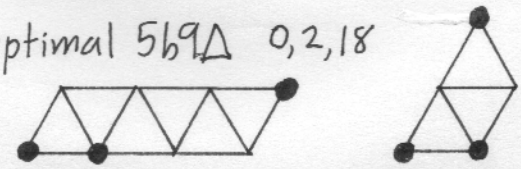
Five Flat-nine Triad (5b9Δ)
0,3,18



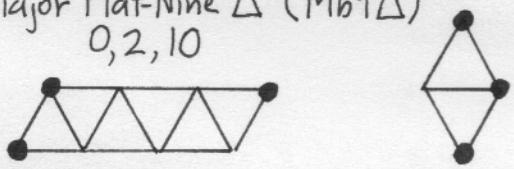
Septimal 5b5Δ 0,16,18



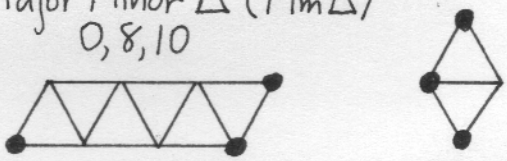
Septimal 5b9Δ 0,2,18



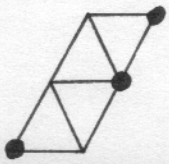
Major Flat-Nine Δ (Mb9Δ)
0,2,10



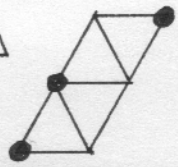
Major Minor Δ (MmΔ)
0,8,10



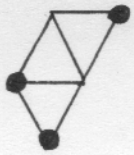
Septimal Mb9Δ
0,3,10



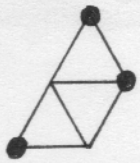
Septimal MmΔ
0,7,10



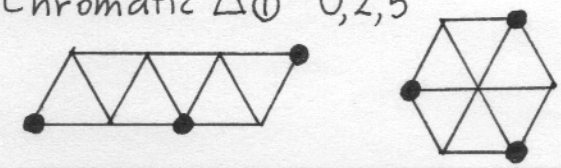
Dissonant Mb9Δ
0,3,11



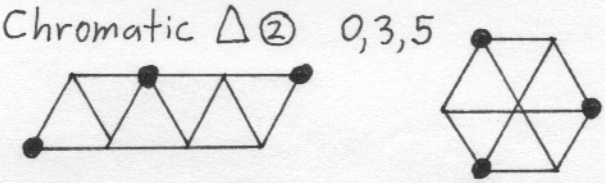
Dissonant MmΔ
0,8,11



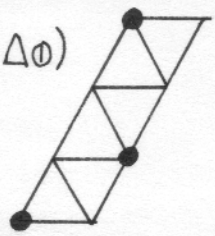
Chromatic Δ① 0,2,5



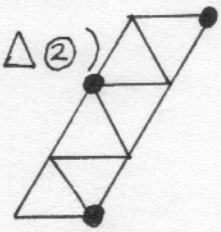
Chromatic Δ② 0,3,5



Enharmonic Δ① (Enh.Δ①)
0,2,3



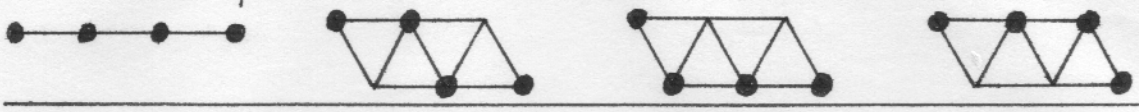
Enharmonic Δ② (Enh.Δ②)
0,1,3



The 31-E.T. Tetrads are divided into two groups, the larger group being chordal, and the smaller group scalar. I have tabulated a relatively large number of triads and tetrads, the most used chordal structures. Pentads and Hexads are "thick" chords or scales. Heptads are generally considered as scales.

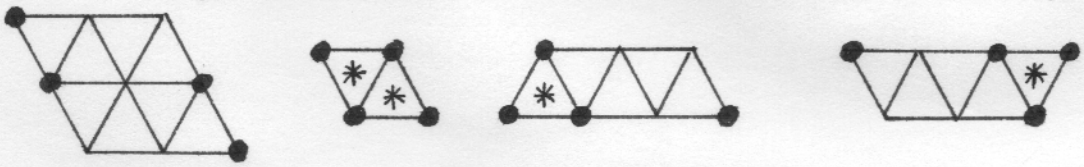
Chordal Tetrads

Suspended Tetrad (Sus □) - Symmetrical 0,5,13,18



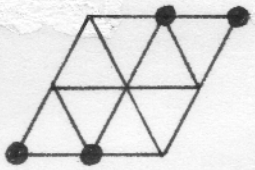
Major Sixth-Minor Seventh Tetrad (M6□-m7□) - Symmetrical

0,10,18,23
8,18,26,0



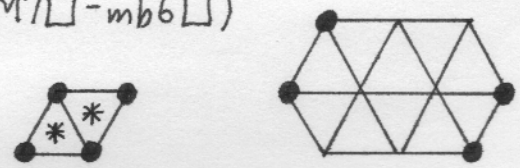
Septimal Major-Sixth, Minor Seventh □

Symmetrical 0,11,18,24
7,18,25,0



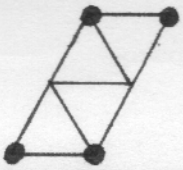
Major-Seven, Minor Flat-six □ (M7□-mb6□)

Symmetrical 0,10,18,28
21,0,8,18

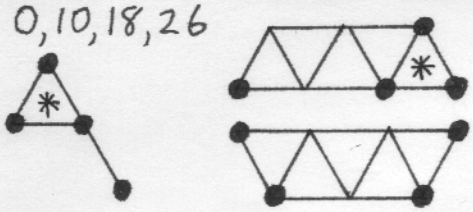


Septimal Major-Seven, Minor Flat-six □

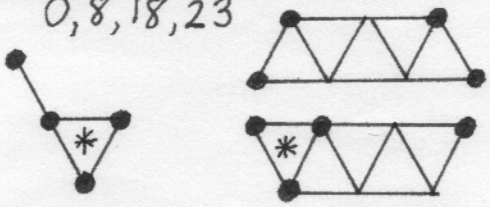
Symmetrical 0,11,18,29
20,0,7,18



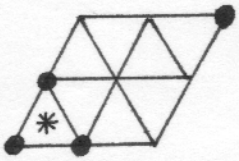
Dominant Seventh Tetrad (Dom7□)
0,10,18,26



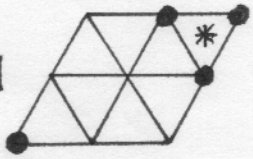
Minor Sixth Tetrad (m6□)
0,8,18,23



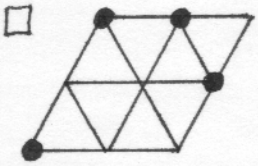
Septimal Dom7□
Harmonic Series □
"German Sixth Chord"
0,10,18,25



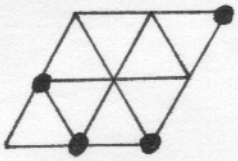
Septimal m6□
Subharmonic Series □
0,8,18,24



Dissonant Dom7□
0,11,18,26



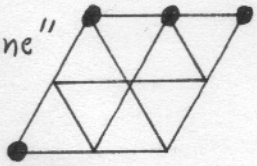
Dissonant m6□
0,7,18,23



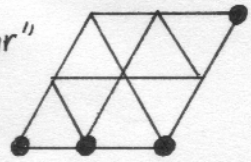
Major Triad "add nine" (M9□)
0, 5, 10, 18

Minor Triad "add four" (eleven) m11□
0, 8, 13, 18

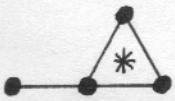
Septimal Major △ "add nine"
0, 5, 11, 18



Septimal Minor △ "add four"
0, 7, 13, 18

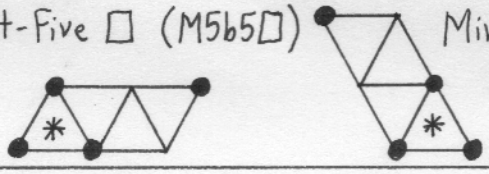


Major △ "add four" (M11□)
0, 10, 13, 18

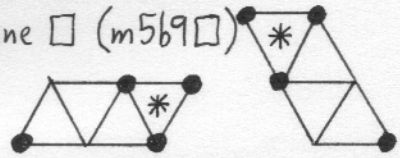


Minor △ "add nine" (m9□)
0, 5, 8, 18

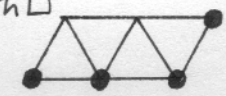
Major Five-Flat-Five □ (M5b5□)
0, 10, 15, 18



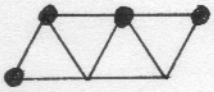
Minor Five-Flat-Nine □ (m5b9□)
0, 3, 8, 18



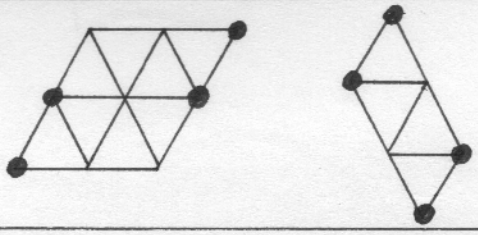
Major Seventh, Suspended Fourth □
(M7s4□) 0, 13, 18, 28



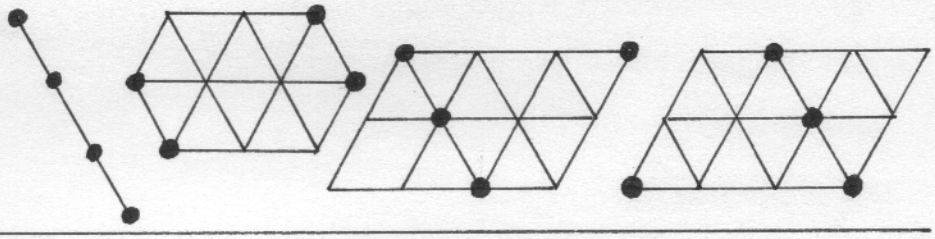
Flat-two, suspended Four □
(b2s4□) 0, 3, 13, 18



Seven-Flat-Five □ (7b5□)
"French Sixth Chord"
Symmetrical 0, 5, 15, 21
26, 0, 10, 16

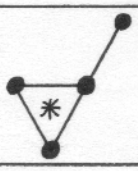


Diminished □ (°□)
Symmetrical
0, 8, 15, 23

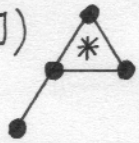


Major-Minor Tetrad (Mm□)
Symmetrical 0, 8, 10, 18

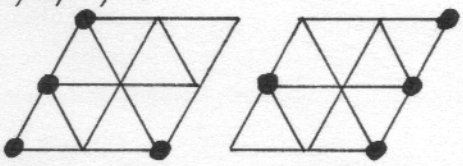
Minor Natural-Seven □ (m♮7□)
0, 8, 18, 28



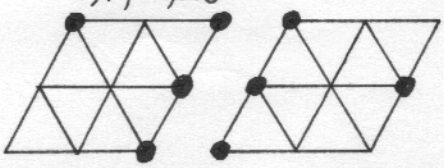
Major Flat-six □ (Mb6□)
0, 10, 18, 21



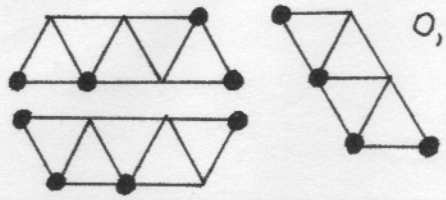
Seven-sharp-five □① (7#5□)
0, 10, 21, 26



Seven-sharp-five □② (7#5□)
0, 11, 21, 26

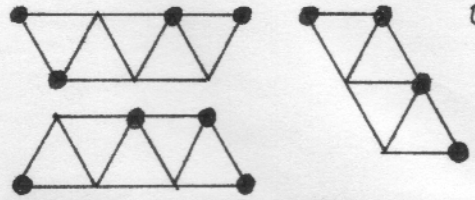


Six, Five-Flat-Five □ (6,5b5□)



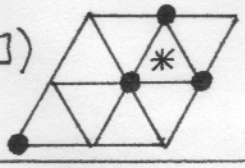
0,15,18,23

Seven, Five-Flat-nine □ (7,5b9□)

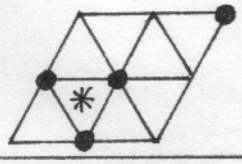


0,3,18,26

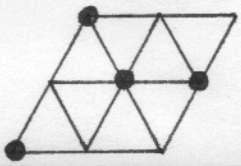
Major Flat-nine □ (Mb9□)
0,3,10,18



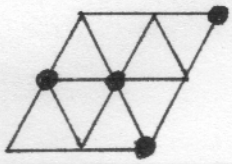
Minor Five-Flat-Five □
(m5b5□) 0,8,15,18



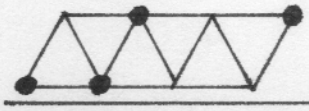
Six, Five-Flat-nine □
(6,5b9□)
0,3,18,23

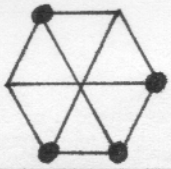


Seven, Five-Flat-five □
(7,5b5□)
0,15,18,26

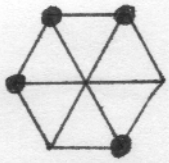


Five, Major-Seven, Flat-nine □
(5,M7,b9□)
0,2,18,28

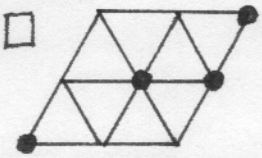




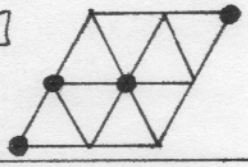
Flat-six, Five-Flat-Five □
(b6,5b5□)
0,16,18,21



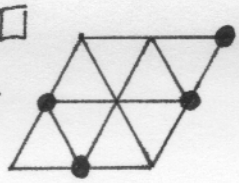
Five, Major-Seven, Flat-nine □
0,3,18,28



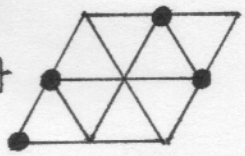
Flat-six, Five, Flat-five □
0,15,18,21



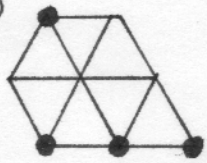
Dominant Seven add Flat-nine □
(Dom7,b9□)
Diminished scalar segment
0,3,10,26



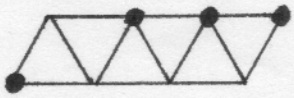
Major-minor three, Flat-Five □
(Mm3,b5□)
Diminished scalar segment
0,7,10,15

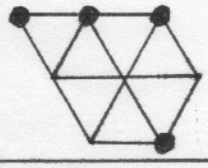


Five, Flat-Five, Suspended Four □①
(5,b5s4□) 0,13,15,18

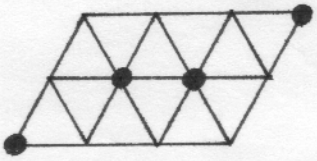


Five, Flat-Five, Suspended four □②
(5,b5s4□) 0,13,16,18





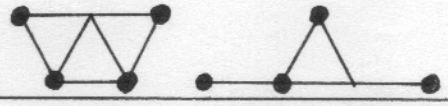
Five, Flat-Five, Flat-nine Tetrad – Symmetrical
(5,b5,b9□) 0,3,15,18



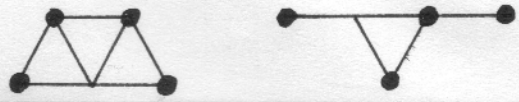
The scalar tetrads include the patterns which were traditionally called "tetrachords."

Scalar Tetrads

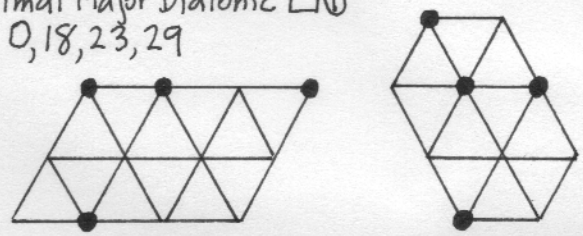
Major Diatonic Tetrad (Mdiat□)
0, 18, 23, 28



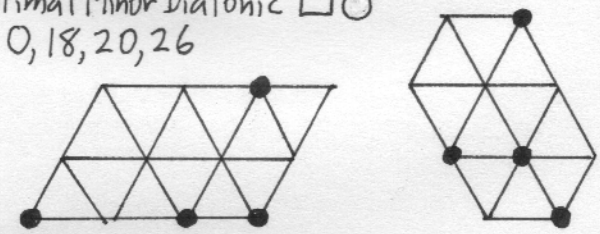
Minor Diatonic Tetrad (mdiat□)
0, 18, 21, 26



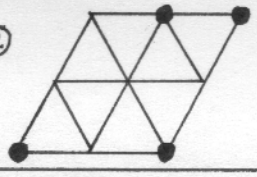
Septimal Major Diatonic □①
0, 18, 23, 29



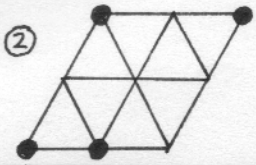
Septimal Minor Diatonic □○
0, 18, 20, 26



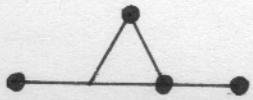
Septimal Major Diat. □②
0, 18, 24, 29

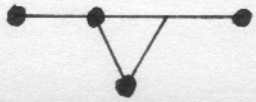


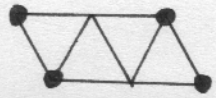
Septimal Minor Diat. □②
0, 18, 20, 25

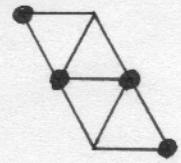


Five, Six, Seven Scalar □ (5,6,7□) - Symmetrical 0, 18, 23, 26

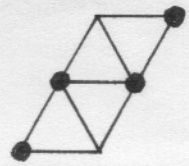




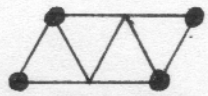




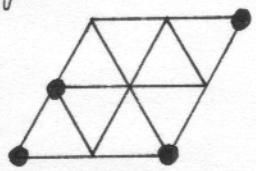
Major-Seven, Flat-six Scalar Tetrad (M7, b6□)
Symmetrical 0, 18, 21, 28



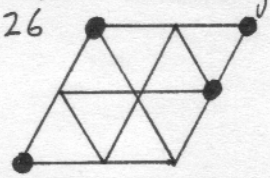
Whole-tone Scalar Tetrad (W.T.□)
Symmetrical 0, 5, 10, 15



Septimal Whole-tone Scalar Tetrad①
(Harmonic Series segment)
0, 5, 10, 25

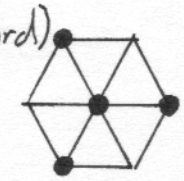


Septimal Whole-tone Scalar Tetrad②
(Sub-harmonic Series segment)
0, 6, 21, 26

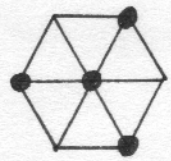


Major-seven, Five-Flat-Five Scalar □ (M7, 5b5□)
0, 10, 13, 28 Symmetrical

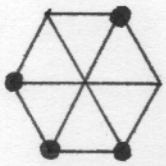
Five, Six, Flat-six Scalar □
(Classical Gk. Chromatic tetrachord)
(5, 6b6□)
0, 18, 21, 23



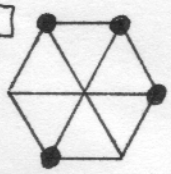
Five, Major-minor Seven □
(5, Mm7□)
0, 18, 26, 28



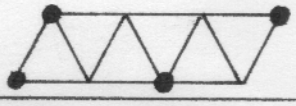
Septimal Five, Six, Flat-six □
0, 18, 20, 23

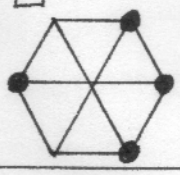


Septimal Five, Major-minor Seven □
0, 18, 26, 29

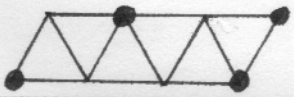


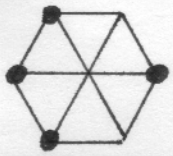
Flat-two, two, three Cluster □
(b2, 2, 3 □) 0, 2, 5, 10



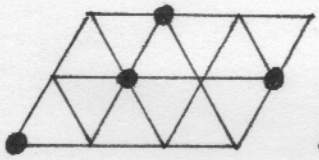


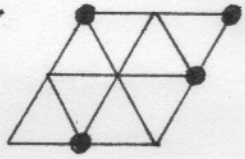
Two, Major-minor three Cluster □
(2, Mm3 □) 0, 5, 8, 10



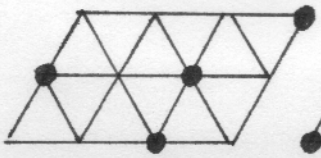


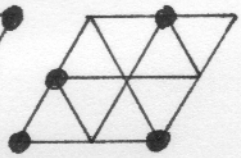
Septimal Flat-two, two, three □ 0, 3, 5, 10



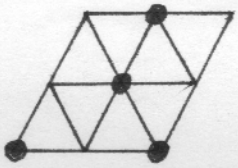


Septimal two, Major-minor three □ 0, 5, 7, 10

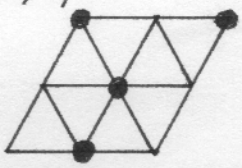




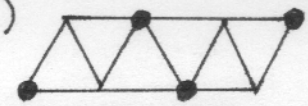
Flat-two, Major-minor three Cluster □ ①
(b2, Mm3 □) 0, 3, 8, 10

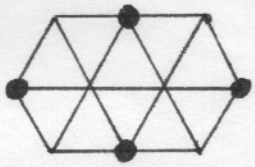


Flat-two, Major-minor three Cluster □ ②
(b2, Mm3 □) 0, 2, 7, 10

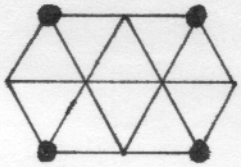


Chromatic Tetrad (Chrom □)
Symmetrical 0, 3, 5, 8

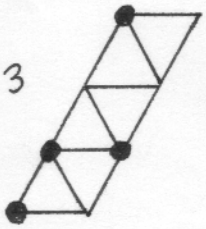




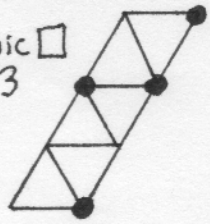
Septimal Chromatic Tetrad
Symmetrical 0, 2, 5, 7



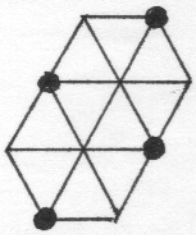
Archytas Enharmonic □
(Arch. Enh. □) 0, 2, 3, 13



Inverse Archytas Enharmonic □
(Inv. Arch. □) 0, 10, 11, 13

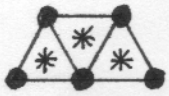


Enharmonic Tetrad (Enh. □)
Symmetrical 0, 2, 3, 5

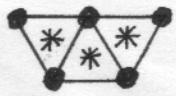


The Pentads

Major-seven, nine Pentad
Ptolemaic Pentad ①
0, 5, 10, 18, 28

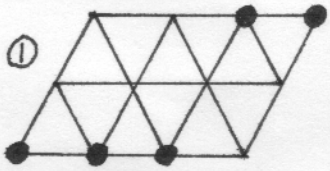


Minor-seven, nine Pentad
Ptolemaic Pentad ②
0, 5, 8, 18, 26

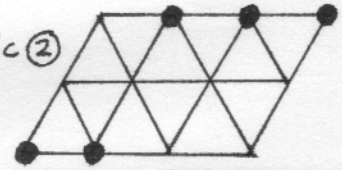


Major-six, nine Pentad – Symmetrical 0, 5, 10, 18, 23
Pentatonic Scales

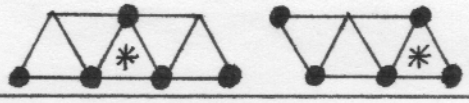
Septimal Pentatonic ①
0, 6, 11, 18, 24



Septimal Pentatonic ②
0, 5, 11, 18, 24

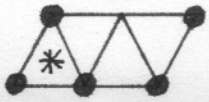


Major Scalar Pentad (Msc.)
0, 5, 10, 13, 18



Minor Scalar Pentad (m sc.)
0, 5, 8, 13, 18

Lydian Scalar Pentad
0, 5, 10, 15, 18

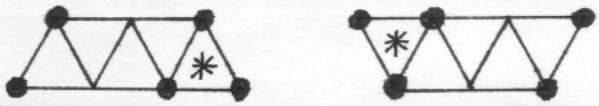


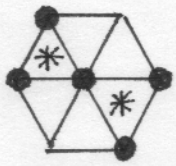
Phrygian Scalar Pentad
0, 3, 8, 13, 18

Olympus Gk. Scale
Chinese Sc., Koto
0, 5, 8, 18, 21

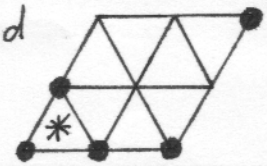
Major-Seven, eleven Pentad
"Balinese" scale
0, 10, 13, 18, 28

Dominant Seven, Nine Pentad – Symmetrical
Minor-sixth, eleven Pentad
0, 5, 10, 18, 26
13, 18, 23, 0, 8

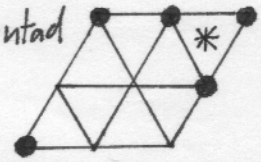




Harmonic Series Pentad
0, 5, 10, 18, 25

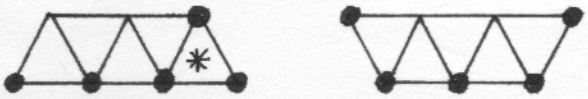


Sub-harmonic Series Pentad
0, 8, 13, 18, 24



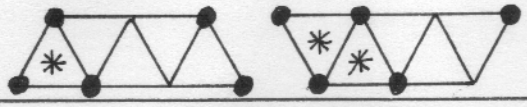
Dominant Scalar Pentad
Symmetrical
0, 5, 10, 13, 28

Suspended □ "add Natural Seventh"
0, 5, 13, 18, 28

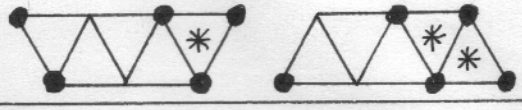


Suspended □ "add Flat-sixth"
0, 5, 13, 18, 21

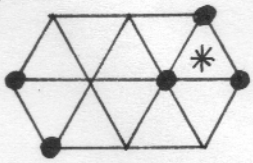
Major-Sixth Tetrad "add sharp-eleven"
Minor-Sixth □ "add flat-seven"
0, 10, 15, 18, 23



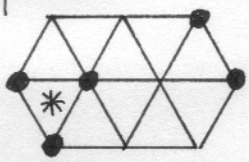
Minor-Seventh □ "add flat-ninth"
Major-sixth □ "add flat-seventh"
0, 3, 8, 18, 26



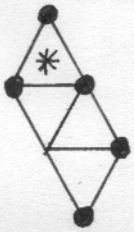
Dominant-seven □ "add flat-nine"
Diminished □ superset
0, 3, 10, 18, 26

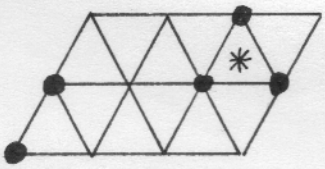


Minor-sixth □ "add sharp-Four"
Diminished □ superset
0, 8, 15, 18, 23

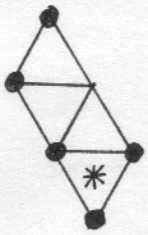


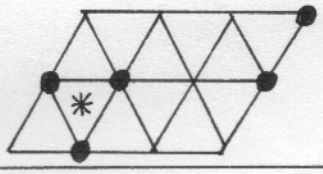
Dominant-seven □ "add Flat-Five"
7b5 □ superset
0, 10, 16, 18, 26



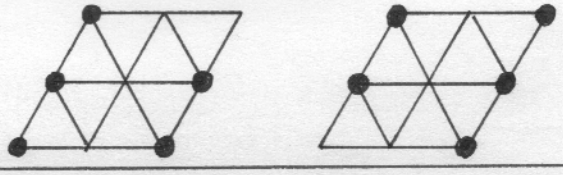


Minor-sixth □ "add flat-nine"
7b5 □ superset
0, 2, 8, 18, 23

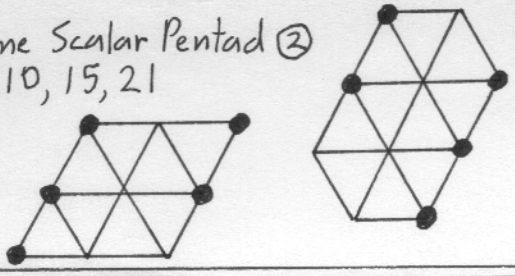




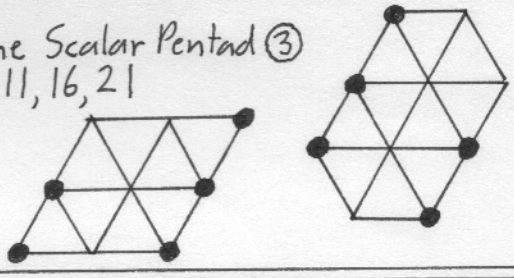
Wholetone Scalar Pentad ①
Symmetrical
0, 5, 10, 15, 20



Wholetone Scalar Pentad ②
0, 5, 10, 15, 21

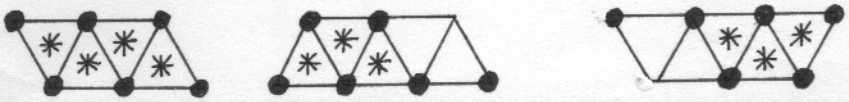


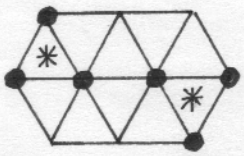
Wholetone Scalar Pentad ③
0, 6, 11, 16, 21



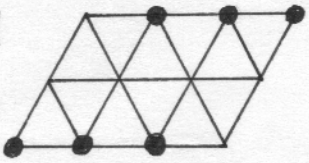
The Hexads

The Hexachord – Symmetrical
Ptolemaic Hexad ①
0, 5, 10, 13, 18, 23

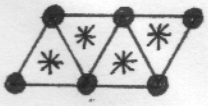




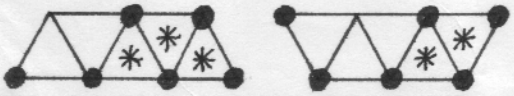
Symmetrical Septimal Hexachord
0, 6, 11, 13, 18, 24



Symmetrical Ptolemaic Hexad ②
M7,11,13 mb6,b7,9 M7,9,#11
0,10,13,18,23,28



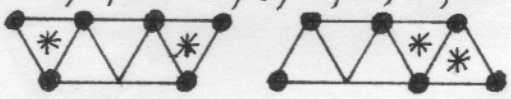
Ptolemaic Heptad Subset
M7,9,11 0,5,10,13,18,28



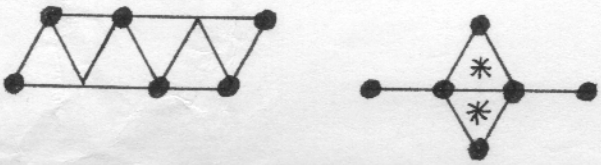
Ptolemaic Heptad Subset
mb6,9,11 0,5,8,13,18,21

Ptolemaic Heptad Subset
Dom 7,9,11 0,5,13,18,23,28

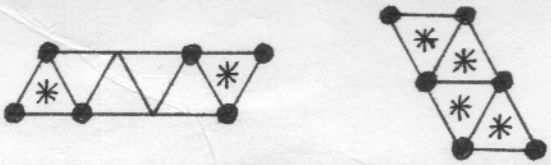
Ptolemaic Heptad Subset
m6,9,11 0,5,13,18,21,26



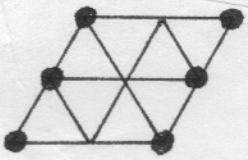
Symmetrical Scalar Hexad
0,5,8,10,13,18



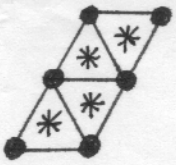
Symmetrical Hexad
M6,b9,#11 m7,b9,#11
0,5,13,15,23,28



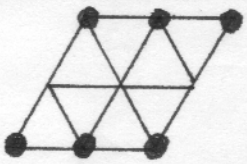
Wholetone Scalar Hexad
Symmetrical 0,5,10,15,20,25



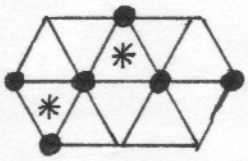
Symmetrical Septimal Hexad
Mm□,b6,M7 0,8,10,18,21,28



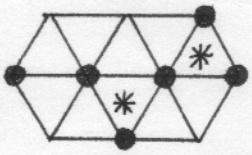
Symmetrical Septimal Hexad
0,2,7,13,18,20



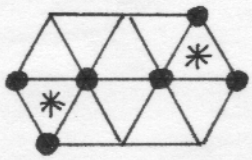
Scalar Hexad
Mb6,9,11 m6,b7,9
0,5,10,13,18,21



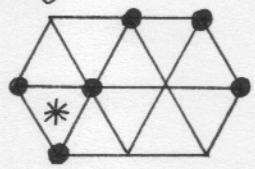
Scalar Hexad
mb7,9,11
Dom 7,11,b6



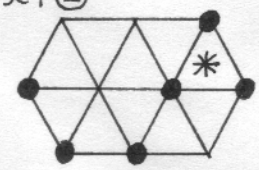
Symmetrical Diminished □ Superset
Dom 7,b9,11 m6,9,#11
0,5,13,18,21,28



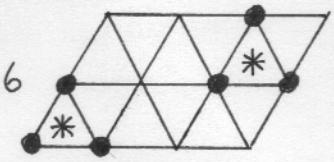
Diminished □ Superset ①
m6, b7, #11
0, 5, 10, 13, 21, 28



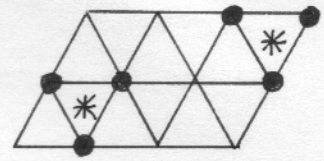
Diminished □ Superset ②
Dom 7, b9, b13
0, 3, 10, 18, 21, 26



Dom 7, b9, #11
0, 3, 10, 16, 18, 26



m6, b9, #11
0, 2, 8, 15, 18, 23

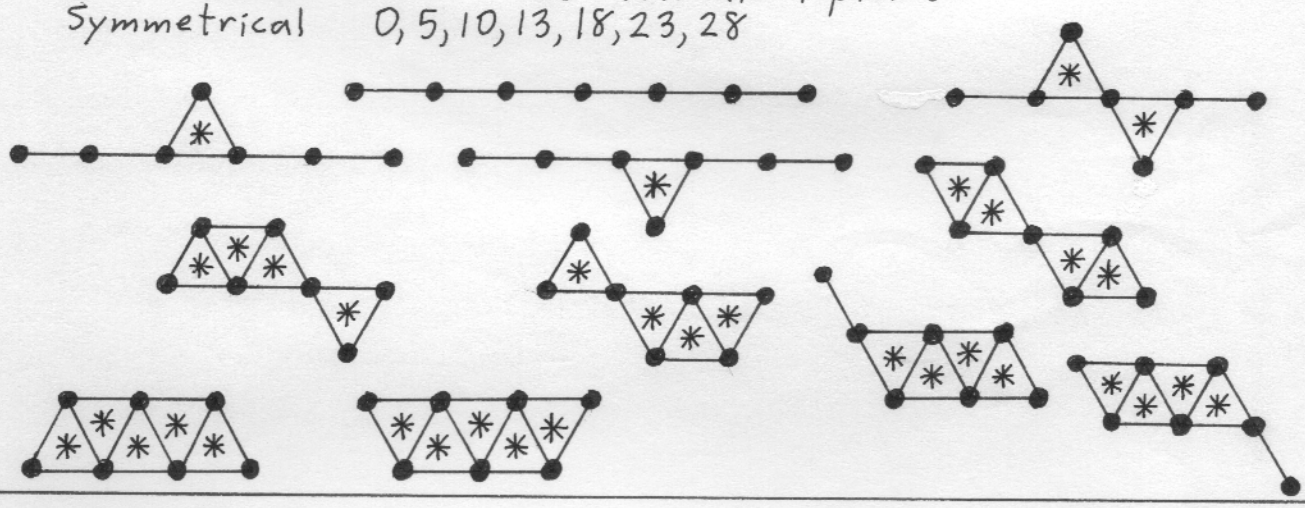


The Heptads

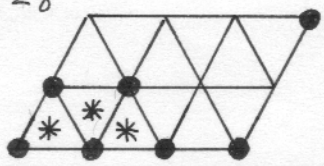
The relatively large number of tabulated heptads reflects the common usage of these patterns as scale types.

The Diatonic Scales – The Ptolemaic Heptads

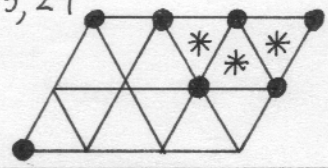
Symmetrical 0, 5, 10, 13, 18, 23, 28



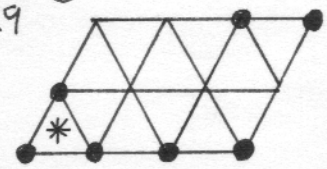
Septimal Diatonic Scale ①
0, 5, 10, 12, 18, 23, 28



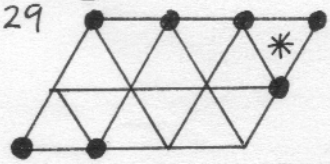
Septimal Diatonic Scale ②
0, 5, 10, 13, 18, 23, 29



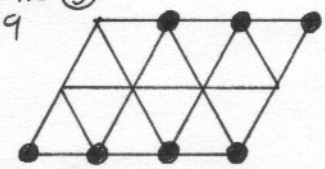
Septimal Diatonic Scale ③
0, 6, 11, 13, 19, 24, 29



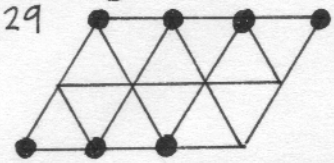
Septimal Diatonic Scale ④
0, 5, 11, 13, 18, 23, 29



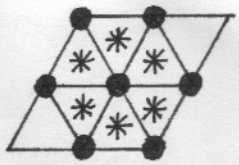
Septimal Diatonic Scale ⑤
0, 6, 11, 13, 18, 24, 29



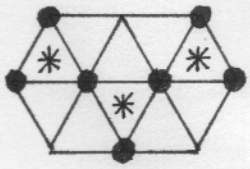
Septimal Diatonic Scale ⑥
0, 5, 11, 13, 18, 24, 29

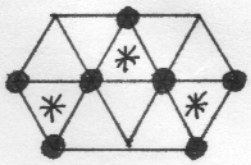


Ellis' Harmonic Heptad
Core chromatic pattern
Symmetrical
0, 8, 10, 13, 18, 21, 23

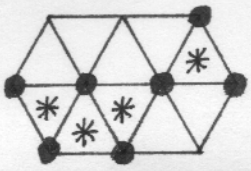


Ascending minor scale
Major-minor scale
0, 5, 10, 13, 18, 21, 26
Symmetrical

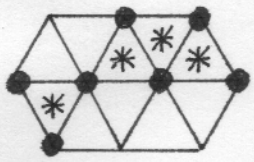




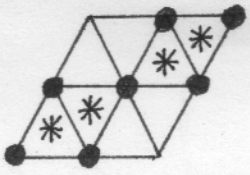
Harmonic Minor Scale
0, 5, 8, 13, 18, 21, 28



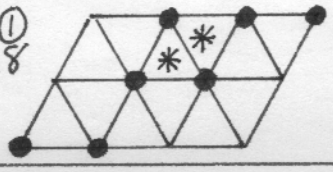
Major Flat-six scale
0, 5, 10, 13, 18, 21, 28



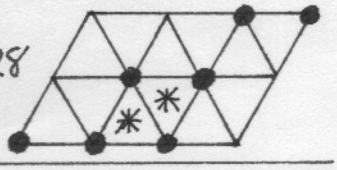
Bhairav-type scale
Hungarian Gypsy scale
Symmetrical
0, 3, 10, 13, 18, 21, 28



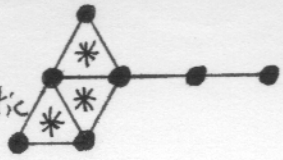
Purvi-type scale ①
0, 3, 10, 15, 18, 21, 28

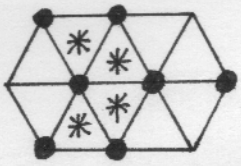


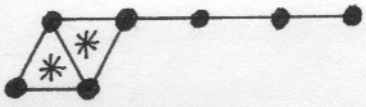
Todi-type scale ①
0, 3, 8, 15, 18, 21, 28



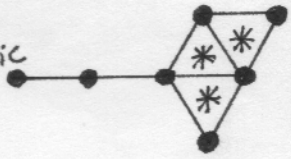
Purvi-type scale ②
0, 2, 10, 15, 18, 20, 28
Greek Didymus Chromatic

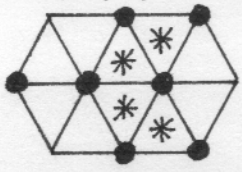


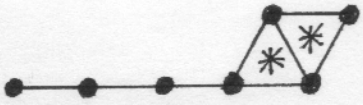




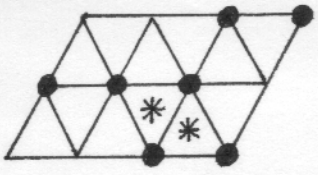
Todi-type scale ②
Inverse Didymus Chromatic
0, 3, 8, 16, 18, 21, 29

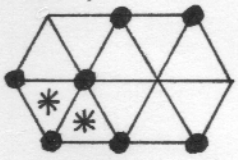




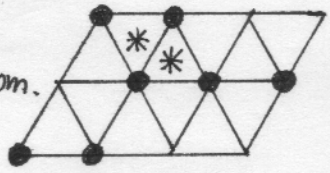


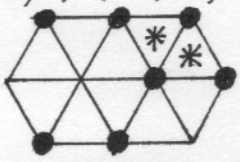
Purvi-type scale ③
Ptolemy 5of Chromatic
0, 3, 11, 16, 18, 21, 29



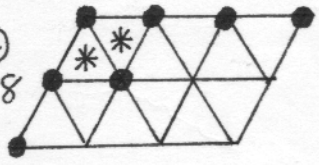


Todi-type scale ③
Inverse Ptolemy 5of Chrom.
0, 2, 7, 15, 18, 20, 28

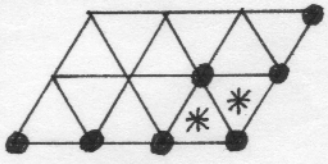




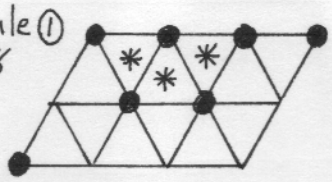
Purvi-type Scale ④
0, 2, 10, 15, 18, 21, 28



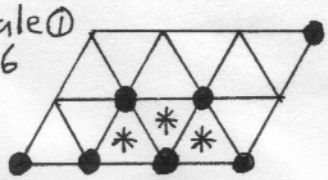
Todi-type Scale ④
0, 3, 8, 16, 18, 21, 28



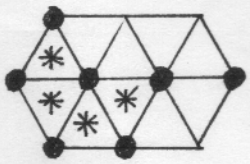
Marva-type scale ①
0,3,10,15,18,23,28

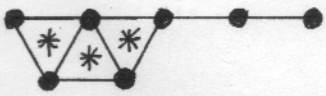


Oriental-type scale ①
0,3,8,15,18,21,26

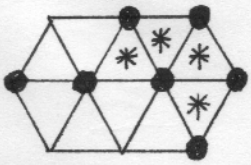


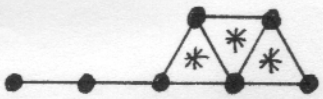
Marva-type scale ②
0,2,10,15,18,23,28



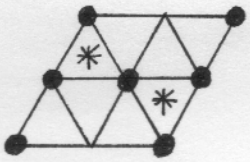


Oriental-type scale ②
0,3,8,16,18,21,26

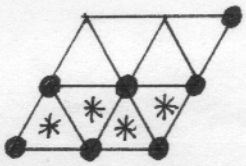




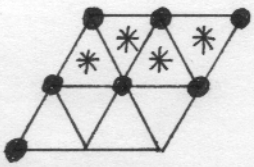
Augmented Heptad - symmetrical
0,3,8,13,18,23,28



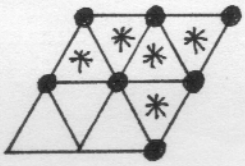
Neapolitan scale ①
0,3,8,13,18,21,28



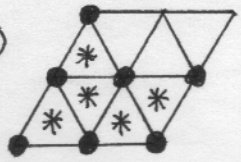
Inverse Neap. scale ①
0,3,10,13,18,23,28



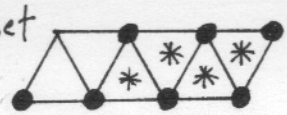
Neapolitan scale ②
0,3,8,13,18,21,29



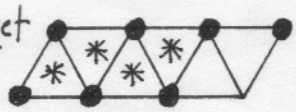
Inverse Neap. scale ②
0,2,10,13,18,23,28



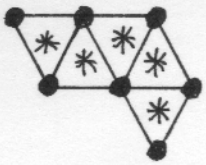
Ptolemaic Ogdoad subset
0,5,10,13,15,18,28



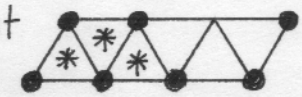
Ptolemaic Ogdoad Subset
0,10,13,15,18,23,28

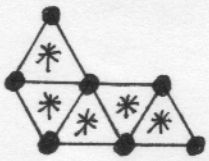


Ptolemaic Ogdoad Subset
0,5,8,10,13,18,23

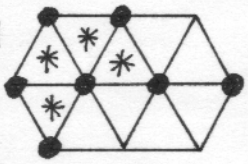


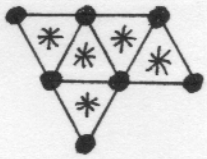
Ptolemaic Ogdoad Subset
0,5,10,13,15,18,23

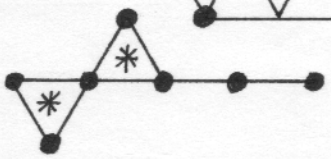


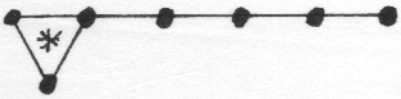


Chromatic-type Heptad
0,5,8,10,18,23,28

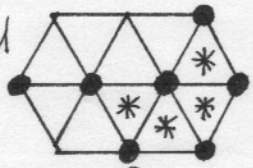


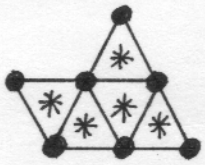


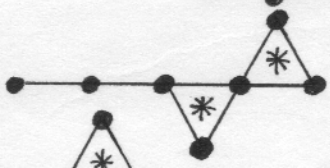




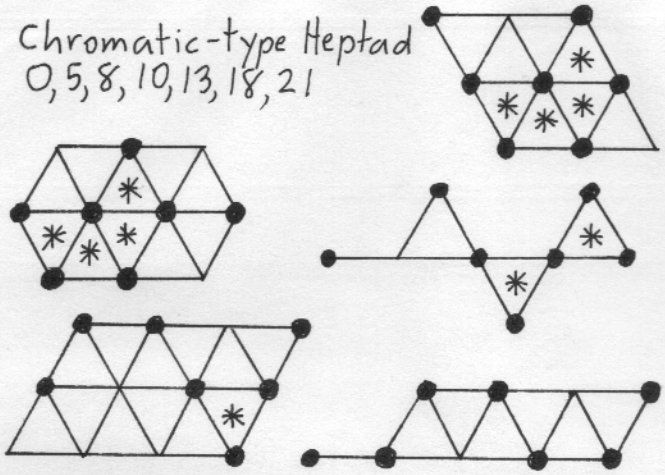
Chromatic-type Heptad
0,5,10,18,20,23,28



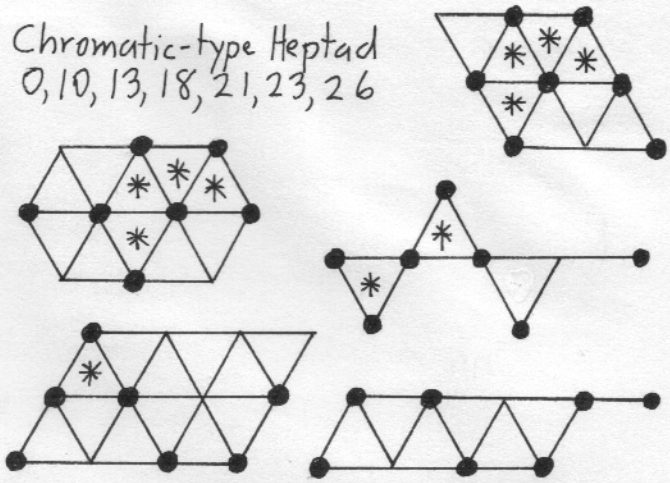




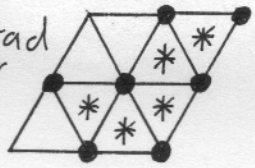
Chromatic-type Heptad
0, 5, 8, 10, 13, 18, 21



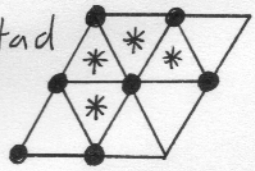
Chromatic-type Heptad
0, 10, 13, 18, 21, 23, 26



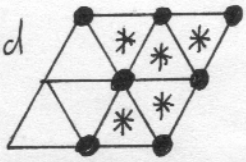
Chromatic-genus Heptad
0, 8, 10, 13, 18, 21, 28



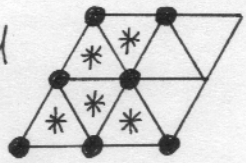
Chromatic-genus Heptad
0, 3, 10, 13, 18, 21, 23



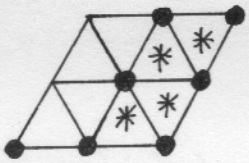
Chromatic-genus Heptad
0, 8, 11, 13, 18, 21, 29



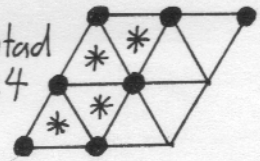
Chromatic-genus Heptad
0, 2, 10, 13, 18, 20, 23



Chromatic-genus Heptad
0, 7, 10, 13, 18, 20, 28

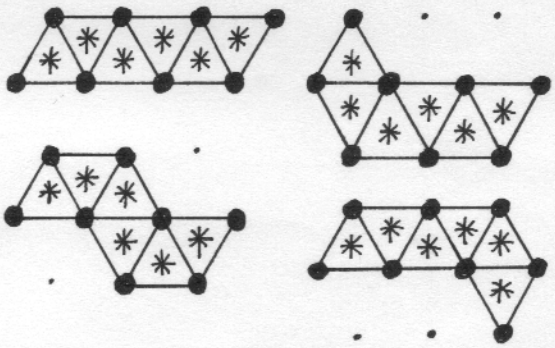


Chromatic-genus Heptad
0, 3, 11, 13, 18, 21, 24

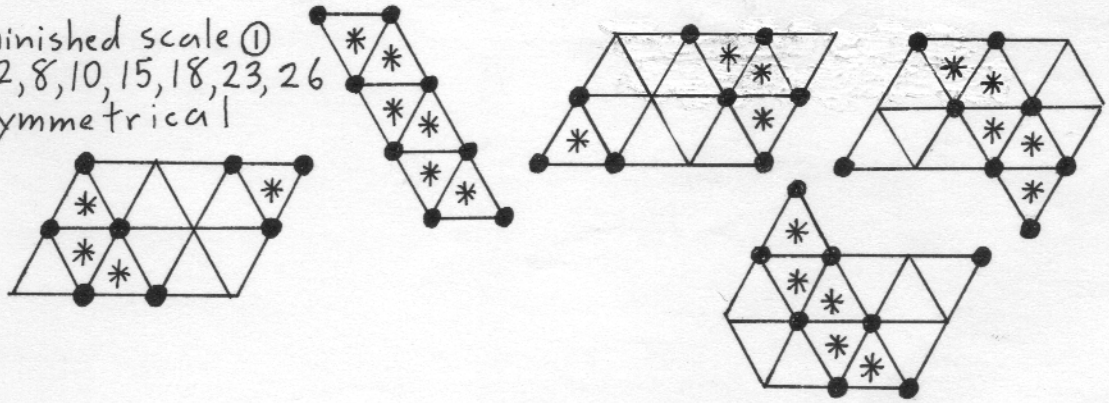


A few prominant ogdoads

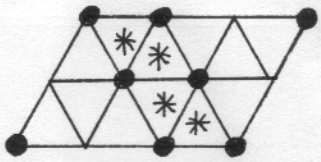
Ptolemaic Ogdoad
Diatonic Ogdoad
Symmetrical
0, 5, 10, 13, 15, 18, 23, 28

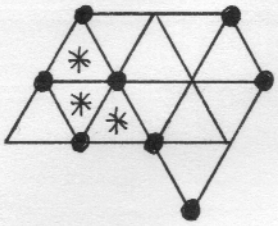


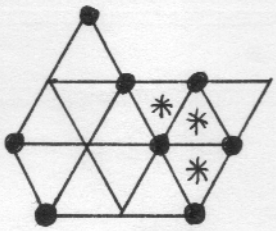
Diminished scale ①
0, 2, 8, 10, 15, 18, 23, 26
Symmetrical

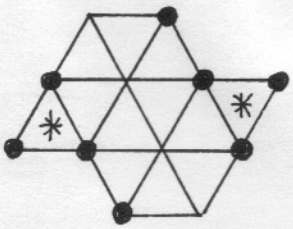


Diminished scale ②
0, 3, 8, 10, 15, 18, 23, 26
Symmetrical

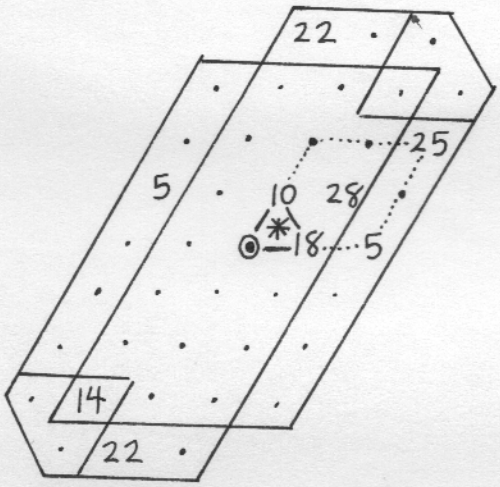




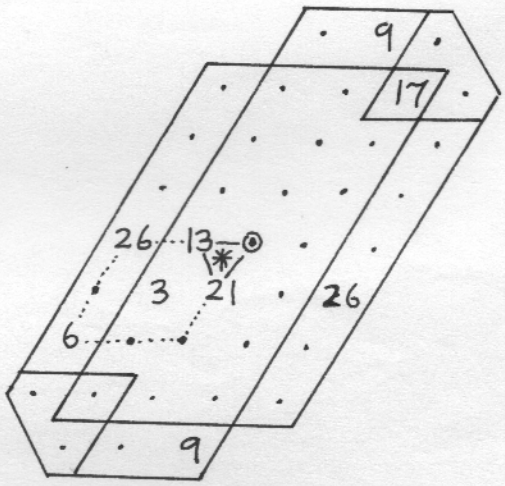




Harmonic Series Ogdoad
(8th to 16th harmonics)
0, 5, 10, 14, 18, 22, 25, 28

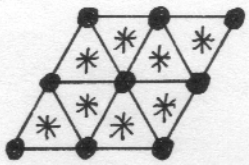


Sub-harmonic Series Ogdoad
(8th to 16th sub-harmonics)
0, 3, 6, 9, 13, 17, 21, 26

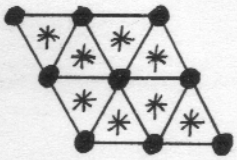


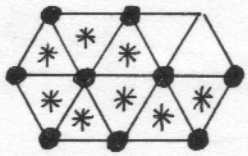
<u>A few prominant higher classes</u>:

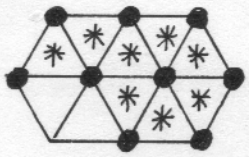
Harmonic ennead
Core chromatic (septimal) pattern
Symmetrical
0, 3, 8, 10, 13, 18, 21, 23, 28



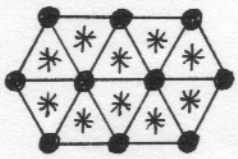
Enneachord - symmetrical, 0, 5, 8, 10, 13, 18, 21, 23, 26



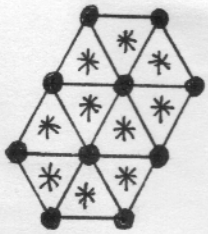




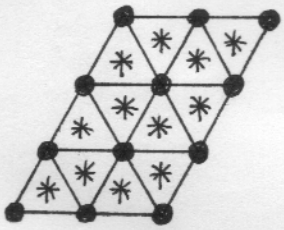
The Harmonic Decad (Ellis)
Ellis' "Unit of harmony"
Essential core of chromatic scale
Symmetrical
0, 5, 8, 10, 13, 18, 21, 23, 26, 28



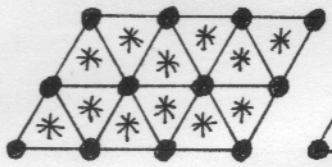
Core enharmonic decad
Symmetrical
0, 2, 8, 10, 13, 18, 20, 21, 23, 28

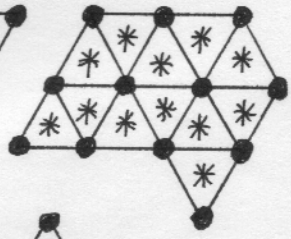


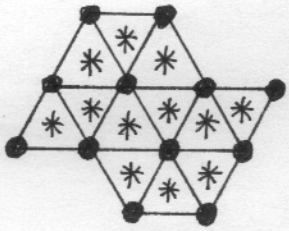
Enharmonic dodecad
Symmetrical
0, 2, 3, 7, 8, 10, 13, 18, 20, 21, 23, 28

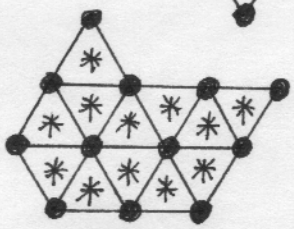


Harmonic dodecad
Chromatic scale
Symmetrical
0, 3, 5, 8, 10, 13, 15, 18, 21, 23, 26, 28

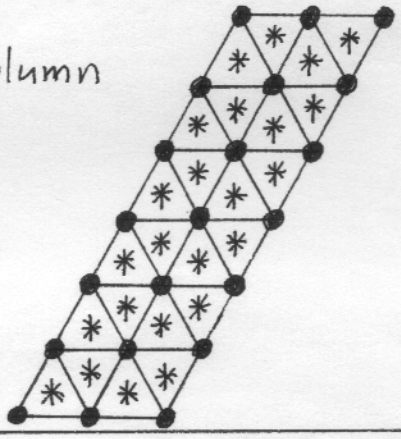




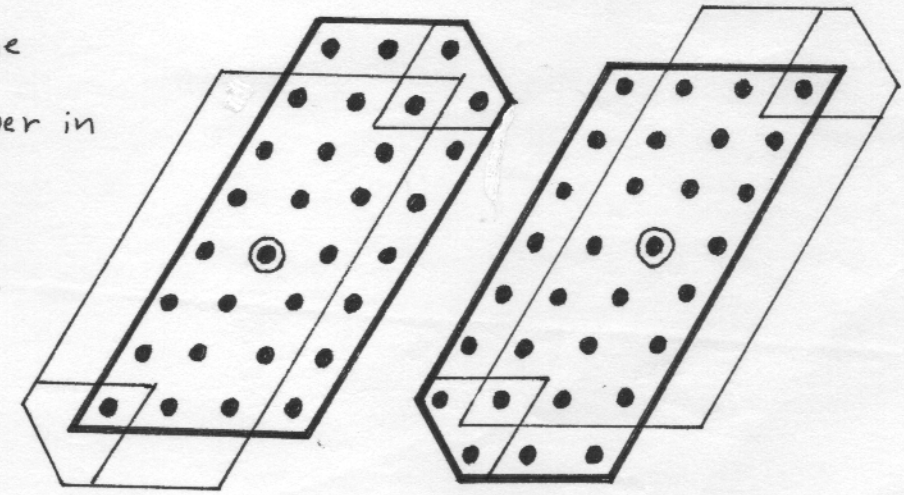




Twenty-one tone central column
Symmetrical



Thirty-one tone
complete set.
Only one member in
this class.
Symmetrical.
Linkage with
the silent set.

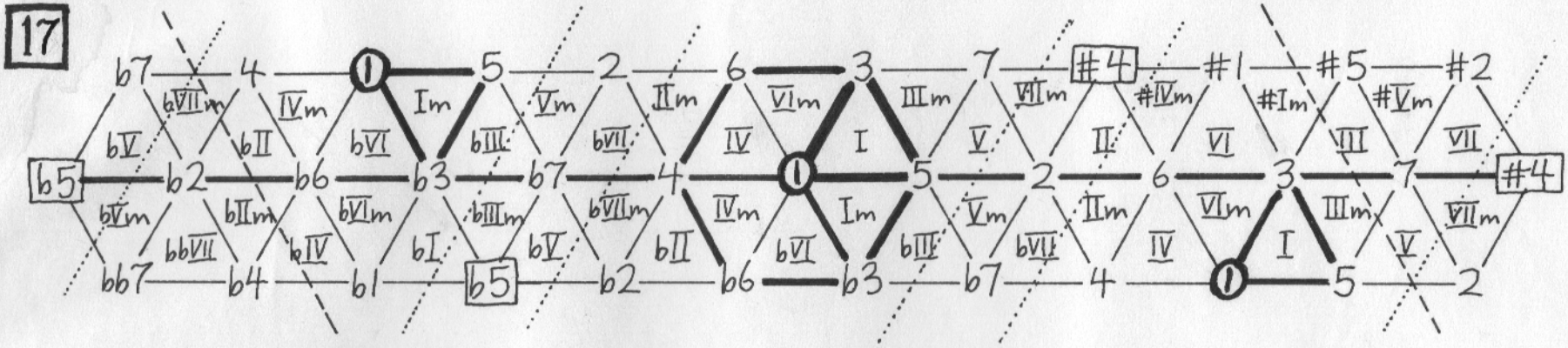


The Function field for 12-E.T.

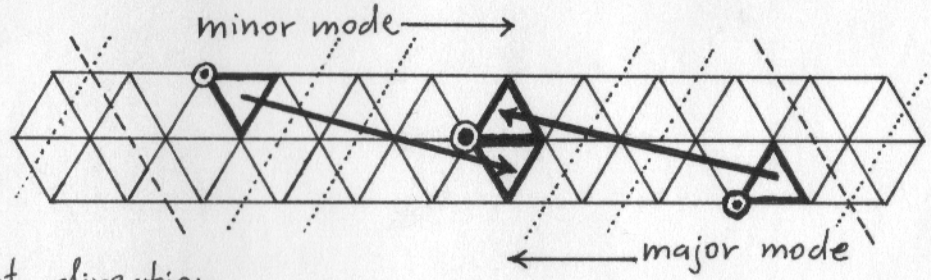
(Chord functions are in Roman Numerals)

b2 (#1)	b3 (#2)		#4 b5	b6 (#5)	b7 (#6)	
1	2	3	4	5	6	7

The twelve pitch functions



Logical progression in the major and minor mode:



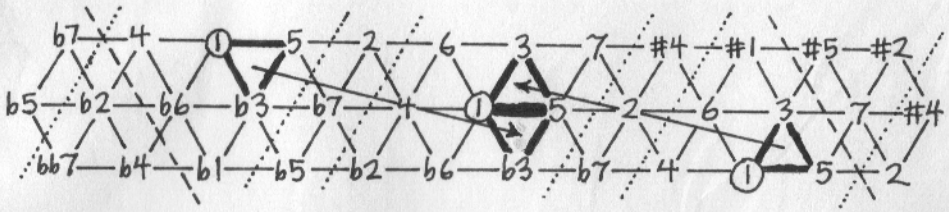
The major mode tends to 'resolve' in the sub-dominant direction. The minor mode tends to 'resolve' in the dominant direction.

Examples of typical chord progressions:

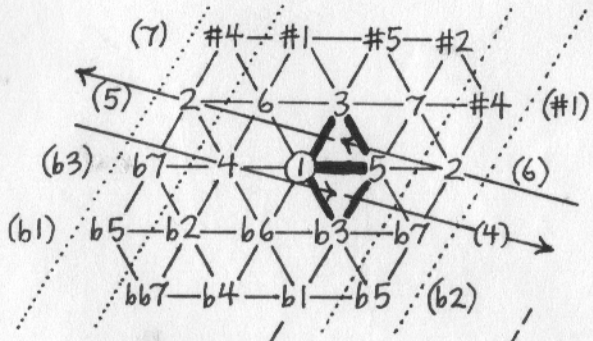
In the major mode	In the minor mode
I – IIm – V – I	Im – bVII – IVm – Im
I – IV – II^7 – V^7 – I	Im – Vm – bVIIm6 – IVm6 – Im
I – III^7 – VIm – V – I	Im – bVIm – bIII – IVm – Im
etc.	etc.

The function field (matrix) for 31-E.T.

The field for 12-E.T. transforms into 31-E.T. The new axis of deviation orients the system along the major third tuning axis rather than the Pythagorean (perfect fifth) axis:

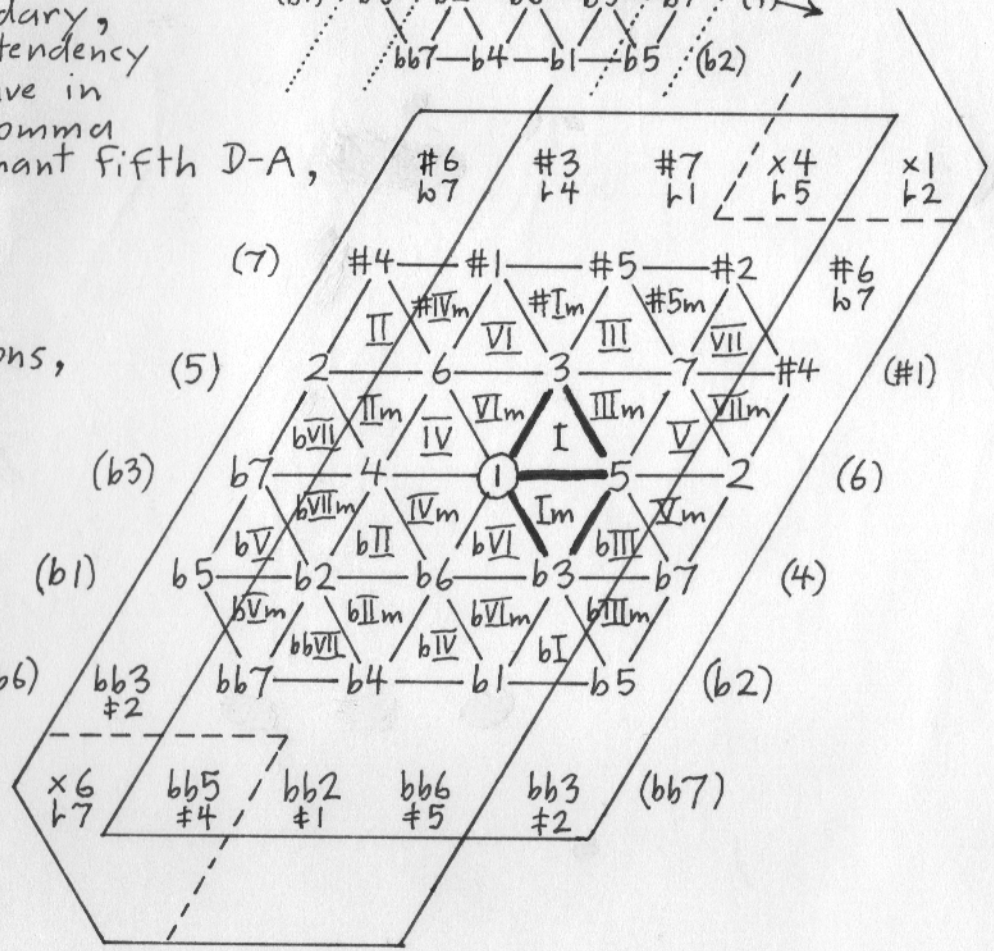


becomes :



Since both systems eliminate the comma boundary, chord progressions in 31-E.T. have a similar tendency of resolution direction to 12-E.T. They behave in identical ways. Systems that preserve the comma boundary (like 53-E.T. or 34-E.T.) and its dissonant fifth D-A, behave in a different manner.

The circle of fifths for 31-E.T. has a wider expansion than the circle for 12-E.T. Thus one can go 'farther afield' in progressions, but such movement to the 'nether zones' around the harmonic antipodes must weaken the gravitational pull of the tonic. Nevertheless, wide ranging progressions are possible within the 'key centred' musical syntax. For example, consider:



$I - \#\underline{IV}^7 - \#\underline{V}m - \#Im - \underline{II}^7 - \underline{V}^7 - I$

$I - \#\underline{II}m - \underline{III}^7 - \underline{VI}m - \underline{IV} - \underline{V}^7 - I$

$Im - bb\underline{VII} - b\underline{VI}m6 - b\underline{III} - \underline{V}m - \underline{IV}m6 - Im$